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Editors: Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi and Eszter Kónya

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Preface: CERME13 in Budapest, and beyond

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The 13th Congress of the European Society for Research in Mathematics Education – CERME13, for short – was a memorable event in many ways. It was the first CERME held “on site” since CERME 11 in Utrecht in February 2019, almost 4½ years before. After the outbreak of the COVID pandemic in early 2020, CERME12 was postponed to 2022 and was finally held online from Bozen-Bolzano in February 2022, as explained in the preface to the proceedings of that conference. At that time, it was still not clear whether the pandemic was nearing the end – although, as we now know, it was – and it was decided to defer the dates of CERME13 from February 2023 to July 2023. This way, we avoided the risks of new virus-related lockdowns in early 2023, which in fact did not materialize – and also real inconveniences related to the war in Ukraine, such as the energy crisis in the winter 2022-2023. In addition to these reasons, the new dates of CERME13, almost exactly between CERME12 and CERME14 (to be held in February 2025), allowed for a more realistic timeframe for organising CERME13 and CERME14 (1½ year for each). From February 2025, we will finally be back to the normal rhythm of biannual congresses in February of odd years, after CERME12 and CERME13 were exceptionally held in somewhat modified modalities and dates.

CERME13 took place in Budapest, the beautiful capital of Hungary. The many ways the event was memorable include, besides the reunion of our community after the pandemic, also the first and maybe only CERME ever held in the summer. It cannot be denied that central Europe is much warmer in this season than in the winter, and the hospitality of our Hungarian colleagues was equally warm and efficient. To the local organising committee, led by Csaba Csapodi, Katalin Gosztonyi and Ödön Vancsó, and to the institutions and staff who supported the committee, we extend our warmest thanks on behalf of all participants, for preparing and realising the complex organisation of the congress in a way that fully matched the quality of its former editions. The Lágymányosi campus, with the Faculty of Science buildings of the Eötvös Loránd University, provided the extensive facilities required for both plenary events and the sessions of the thematic working groups. The traditional gala dinner was held in the impressive halls of the Museum of Fine Arts, and added another memorable and very distinguished dimension to the social aspects of the congress.

The scientific programme is evidently the *raison d'être* of any CERME, and the contributions of the participants – papers and posters – form the core material. Unlike many other research conferences, CERME is not focused on presentations, but on in-depth discussions of the contributions. This, indeed, is the secret behind the constantly increasing attraction and success of CERME. Since the very first CERME, the purpose of promoting *communication, cooperation and collaboration* among mathematics education researchers has been implemented by organising the work of the congress in *thematic working groups*. In these groups, the main activity is in-depth discussion of the submitted contributions, with only very brief “reminder presentations” of the texts, which participants in each group are expected to read before attending the congress. At CERME13, no less than 28 such groups

were organised – both new groups, reflecting new specialized areas of research, and groups which have already convened at several previous editions. To organise these groups, teams of coleaders are invited by the international programme committee. Both to every member of these coleader teams, and to the programme committee – led this year by Paul Drijvers and Hanna Palmér – the ERME community is deeply indebted and thankful. We also thank our plenary speakers, László Lovász and Berta Barquero, as well as the plenary panel led by João-Pedro da Ponte, for providing deep and state-of-the-art accounts of research themes that can inspire the community as a whole.

CERME13 received, as almost all previous CERMEs, an unprecedented high number of participants (941), from more nations than ever before (54), see Table 1.

Table 1: The success of CERME13 in numbers: 941 participants from 54 countries

Armenia	1	Germany	165	Romania	1
Australia	7	Greece	20	Serbia	3
Austria	27	Hong Kong	4	Slovakia	14
Belgium	5	Hungary	49	South Africa	2
Brazil	9	Iceland	3	Spain	51
Canada	19	Ireland	13	Sweden	46
Chile	11	Israel	27	Switzerland	14
China	7	Italy	66	Taiwan	2
Costa Rica	1	Japan	9	Thailand	1
Croatia	5	Lithuania	2	Tunisia	3
Cyprus	2	Malta	1	Turkey	46
Czech Republic	17	Mexico	17	Ukraine	1
Denmark	19	Netherlands	32	UAE	1
Egypt	1	New Zealand	5	UK	43
Estonia	4	Norway	66	USA	35
Faroe Islands	1	Philippines	1	Uruguay	1
Finland	6	Poland	1	Vietnam	1
France	24	Portugal	28		

In fact, the number of contributions to CERME congresses has increased almost linearly throughout the history of CERME, to reach a whopping 834 at CERME13. Of these, 757 were finally accepted and presented. Given the capacity of academic venues – roughly 900-1000 participants – and more subtly, of the organisation of a CERME, which depends entirely on voluntary committees and coleader teams – it has become ever clearer that this linear growth cannot continue, even if only one participant for each contribution were to participate. At CERME13, it was only to a small degree possible for co-authors to register, even for experienced scholars who attended all or most of the previous CERME congresses. These dilemmas were discussed at CERME13, and will continue to be worked on in the committees of ERME. Both CERME, and the other venues and supports that ERME offers to scientific exchange and creation by researchers in Europe and beyond, need to be continuously maintained and developed. The enthusiasm and quality of CERME13 promises a bright future for our research field and for the ERME community. See you at CERME14 in Bozen-Bolzano!

Introduction to the proceedings of the thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)

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About CERME13

The thirteenth Congress of European Research in Mathematics Education (CERME13) took place in Budapest, Hungary, from 10 to 14 July 2023. It was organised by the Alfréd Rényi Institute of Mathematics, hosted by the Eötvös Loránd University, and attracted 941 participants. Paul Drijvers and Hanna Palmér were chair and co-chair of the International Programme Committee, which included Federica Ferretti, Eirini Geraniou, Katalin Gosztonyi, Thomas Hausberger (from August 2022), Jeremy Hodgen, Sibel Kazak, Esther Levenson, Bozena Maj-Tatsis, Simon Modeste (until August 2022), Juuso Henrik Nieminen, and Florian Schacht. Csaba Csapodi chaired the Local Organizing Committee, which was co-chaired by Katalin Gosztonyi and Ödön Vancsó.

CERME13 hosted 27 Thematic Working Groups, listed in Table 1. New TWGs were TWG28 on Collaborative Settings in Mathematics Teacher Education and TWG29 on Embodied and Material Studies of Mathematical Behaviour. TWG7 on Mathematics for work, society and personal development: lifelong learning was suppressed this edition, but will reappear at CERME14. TWG11 on algorithms merged with TWG16 on Learning mathematics with technology and other resources. In total, CERME13 had 143 TWG (co-)leaders, who did an extremely valuable job which is key to CERME's success. The conference was preceded by the YERME-day for young researchers in ERME, that attracted 150 participants. Altogether, CERME13 was a success both in academic terms and in social atmosphere, which the European mathematics education research community enjoyed.

Table 1: Overview of the TWG leadership teams

Thematic Working Group	Leader	Co-leaders
TWG1: Argumentation and Proof	Andreas Moutsios-Rentzos	Orly Buchbinder, Nadia Azrou, Fiene Bredow, Dimitrios Deslis, Viviane Durand-Guerrier, David A. Reid, Mei Yang
TWG2: Arithmetic and Number Systems	Pernille Bødtker Sunde	Elisabeth Rathgeb-Schnierer, Renata Carvalho, Ioannis Papadopoulos
TWG3: Algebraic Thinking	Maria Chimoni	Cecilia Kilhamn, Jorunn Reinhardtsen, Luis Radford

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TWG4: Geometry Teaching and Learning	Alik Palatnik	Lina Brunheira, Taro Fujita, Chrysi Papadaki, Petra Surynková
TWG5: Probability and Statistics Education	Caterina Primi	Daniel Frischemeier, Orlando Gonzalez, Sibel Kazak, Aisling Leavy, Martin Andre
TWG6: Applications and Modelling	Jonas Bergman Ärlebäck	Susana Carreira, Britta Eyrih Jessen, Gilbert Greefrath, Yana Lacek, Katrin Vorhölter
TWG8: Affect and the Teaching and Learning of Mathematics	Çiğdem Haser	Inés M Gómez-Chacón, Chiara Andrà, Janina Krawitz, Hanna Viitala
TWG9: Mathematics and Language	Jenni Ingram	Kirstin Erath, Alexander Schüler-Meyer, Máire Ní Ríordáin, Ingólfur Gíslason
TWG10: Diversity and Mathematics Education: Social, Cultural and Political Challenges	Laura Black	Anette Bagger, Sabrina Salazar, Timo Dexel, Juuso Nieminenn
TWG12: History in Mathematics Education	Antonio M. Oller-Marcén	Jenneke Krüger, Tanja Hamann, Bjørn Smestad, Olivera Đokić
TWG13: Early Years Mathematics	Bozena Maj-Tatsis	Camilla Björklund, Dorota Lembrér, Esther Levenson, Andrea Maffia, Marianna Tzekaki
TWG14: University Mathematics Education	Irene Biza	Olov Viirman, Matija Bašić, Ignasi Florensa, Ghislaine Gueudet, Mathilde Hitier, Igor' Kontorovich, Athina Thoma, Megan Wawro
TWG15: Teaching Mathematics with Technology and Other Resources	Ornella Robutti	Bärbel Barzel, Melih Turgut, Gülay Bozkurt, Daniel Thurm
TWG16: Learning Mathematics with Technology and Other Resources	Osama Swidan	Rogier Bos, Eleonora Faggiano, Seçil Yemen Karpuzcu, Simon Modeste, Florian Schacht, Jana Trgalová
TWG17: Theoretical Perspectives and Approaches in Mathematics Education Research	Heather Johnson	Mariam Haspekian, Abdel Seidouvy, Felix Lensing, Cecilie Carlsen Bach
TWG18: Mathematics Teacher Education and Professional Development	Marita Friesen	Tracy Helliwell, Raffaele Casi, Andreas Ebbelind, Janne Fauskanger, Pere Ivars, Macarena Larrain, Libuše Samková
TWG19: Mathematics Teaching and Teacher Practice(s)	Reidar Mosvold	Mark Hoover, Helena Grundén, Siún Nic Mhuirí, Chrysoula Choutou
TWG20: Mathematics Teacher Knowledge, Beliefs and Identity	Francesca Martignone	Miguel Montes, Federica Ferretti, Veronika Hubeňáková, Jimmy Karlsson, Nadia Kennedy, Miguel Ribeiro
TWG21: Assessment in Mathematics Education	Francesca Morselli	Christina Drüke-Noe, Gözde Kaplan-Can, Chiara Giberti, Johanna Rämö
TWG22: Curricular Resources and Task Design in Mathematics Education	Shai Olsher	Annalisa Cusi, Dubravka Glasnović Gracin, Hendrik Van Steenbrugge
TWG23: Implementation of Research Findings in Mathematics Education	Mario Sánchez Aguilar	Boris Koichu, Morten Misfeldt, Linda Marie Ahl

TWG24: Representations in Mathematics Teaching and Learning	Carla Finesilver	Anna Baccaglini-Frank, Elisa Miragliotta, Kate C. O'Brien
TWG25: Inclusive Mathematics Education – Challenges for Students with Special Needs	Petra Scherer	Michael Gaidoschik, Hana Moraová, Helena Roos, Andreas Ulovec
TWG26: Mathematics in the Context of STEM Education	Behiye Ubuz	Michelle Stephan, Clelia Cascella, Nelleke Den Braber
TWG27: The Professional Practices, Preparation and Support of Mathematics Teacher Educators	Ronnie Karsenty	Hilda Borko, Alf Coles, Birte Friedrich-Pöhler, Bettina Rösken-Winter, Stefan Zehetmeier
TWG28: Collaborative Settings in Mathematics Teacher Education	Stéphane Clivaz	Geoff Wake, Paula Gomes, Pernilla Mårtensson, Aoibhinn Ní Shúilleabháin
TWG29: Embodied and material studies of mathematical behaviour	Anna Shvarts	Elizabeth de Freitas, Ricardo Nemirovsky, Christina Krause, Giulia Ferrari

Editorial information

These proceedings are available as a complete volume on the ERME website. Each individual contribution is also available through the HAL open archive, where it can be found through keywords, title, or author name. This has been the practice since CERME9, to increase the visibility of the huge work done in CERME conferences.

After the preface by the president of ERME, Carl Winsløw, and this brief introduction by the proceedings editors, this volume continues with texts on two plenary activities of CERME13: the plenary lecture by Berta Barquero entitled “Mathematical modelling as a research field: Transposition challenges and future directions,” and the report on the Panel on “Bridging the research-practice gap,” chaired by João Pedro da Ponte, with contributions by Mario Sánchez Aguilar, Nad’a Vondrová, Stefan Zehetmeier, Sarah Seleznyov, and Jorryt van Bommel.

The biggest part of these proceedings consists of the sections corresponding to the TWGs. These sections start with an introduction by the TWG leadership team. According to the team’s choice, this is a 4-page introduction presenting the contributions, or an extended 8-page one, which proposes an additional analysis of the current research on the theme of the TWG, and perspectives for the future. Next, the section consists of the paper and poster contributions – in alphabetical order by first author’s name.

These CERME13 proceedings result from a collaboration, involving the CERME13 IPC and LOC, the TWG leaders and co-leaders. Particular thanks are due to Nathalie Kuijpers for her editorial work. We warmly thank all these people for their involvement, and hope that this volume will contribute to the development of mathematics education research in Europe and beyond.