Moving mathematically

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Over the last twenty years, mathematics education research has become increasingly interested in how the body interacts with students' thinking and knowing. However, researchers in this field often theorize the body in diverse ways. From Ingold's post-humanist perspective, humans live in animate bodies which cannot be separated from their thinking and knowing. Movement, for Ingold, is not a support for, or an expression of, thinking, rather human bodies think in movement. This paper studies a small group of students, as they engage with a mathematical task, to investigate students' spontaneous mathematical thinking in movement. As this study illustrates, students' spontaneous thinking in movement can offer new and valuable insights into students' mathematical knowing. These findings suggest mathematics educators may need to reevaluate what might be considered students' mathematical thinking in research and in the classroom.

Keywords: Thinking in movement, mathematics education, embodiment, post-humanism.

Within mathematics education, the links between the body and mathematical thinking are an increasing focus of research. Mathematical education researchers employ a variety of theoretical perspectives to investigate movement and thinking. Although these theories generally reject the historical western separation of mind and body in cognition, they often theorize the body from a variety of different perspectives. Recruiting Tim Ingold's (2013) post humanist theory of making, this paper investigates students' spontaneous movement and thinking. For Ingold, new things emerge from the correspondence of animate human and non-human material flows. Following Maxine Sheets-Johnstone (2011), Ingold argues thinking and movement are inseparable: animate human bodies "think in movement" (Sheets-Johnstone, p. 451).

This paper is part of research for a doctoral thesis which explores students' activities as they engage with a mathematical problem task. As part of the wider thesis, this paper investigates a different fragment from, but the same group and session as, Gandell & Maheux (2019). In order to include all aspects of movement, space, body and dynamic qualities, Laban's movement elements are employed as a movement framework for this study. The aim of this paper is to investigate students' mathematical thinking in movement during a mathematical task. The paper begins with a brief background into the research and theories underpinning the research, describes the research design and movement framework, analyses a small fragment of student activity, and ends by discussing the significance of the findings.

Background

Over the last twenty years, embodiment research, investigating the role of students' and lecturers' bodies in and as their knowing, has gained traction in mathematics education (Abrahamson et al., 2020; Roth, 2015). Although challenging the long held paradigm, separating mind and body in cognition, embodiment research encompasses a variety of, sometimes conflicting, theories. In addition, many of these theories, which include sensuous cognition, cognitive psychological frameworks (for example, grounded blends, conceptual metaphors, and gesture research), enactivism, and inclusive materialism, hold a variety of diverse perspectives on the body.

As Ingold (2013) and Sheets-Johnstone (2011) explain, embodiment research often appears to continue the divide between mind and body. For example, in some embodiment research, students' movement may be dictated by specified tools (often digital tools), or students may be required to use preplanned movements, to produce a predetermined output (Abrahamson et al., 2020). In this way movement may be conceived as a demonstration of concepts held in the mind with the body positioned as an instrument of that intellectual knowing mind (Roth, 2011). For Ingold and Sheets-Johnstone, however, humans are tactile-kinaesthetic beings, not reified minds enclosed in a body package. As primarily tactile-kinaesthetic beings, humans use their bodies, from before birth and before language, to explore and come to know the world (Sheets-Johnstone). From Ingold's post-humanist perspective, humans come to know as human and non-human material flows correspond in an ongoing, ever changing, "dance of animacy" (p.101). In this way knowledge emerges from the flows of animate materials answering to each other, rather than by building representations of the world in ordered steps. So that, as Sheets-Johnstone explains, humans experience "thinking in movement" (p. 451), not by having thoughts in the mind expressed as movement, nor by having movements creating thinking in the mind, but the movement itself is the thinking.

Research design

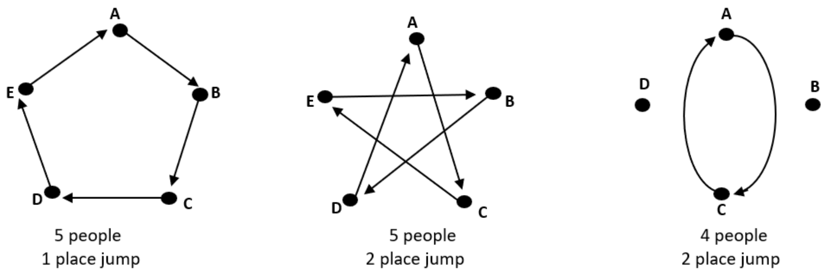
To investigate students' mathematical thinking in movement, this paper follows the flows of materials forward, as Ingold (2013) suggests. Working together, students offer their actions, including their movements and verbalisations, to each other as indications of their knowing (Roth, 2016). Any actions, made available to others, can then be used by researchers as a representation of the students' knowing, without resorting to presupposing students' intentions or thoughts (Roth). By micro-analyzing students' verbalizations and movements, this analysis is concerned only with the actions the students provide for each other, as they engage with a mathematical task. Thus, students performed actions are taken as their knowing, rather than guessing at students' intentions.

Movement framework

Humans, with similar bodies, share an understanding and recognition of how bodies move and how they feel as they move (Sheets-Johnstone, 2011). This shared social understanding of movement is essential for survival and reproduction in any animate social species. Typically, mathematics education research usually considers how the body moves through space. Inherent in any movement, however, are dynamic qualities which evoke sensations for both the performer and for the observer (Laban, 2011/1966, Sheets-Johnstone). Consider how stomping heavily feels different to running lightly and how these movements might feel different for an observer. During the 1930's Laban developed detailed framework of movement elements describing both quantitative (body and space) and dynamic qualitative (effort actions) of movement. Laban's elements are now used in a variety of fields including the arts, industry, psychology research and computer interface technology. Over time Laban's framework has been adapted, however, three elements are generally described: body (parts and actions); space (reach and direction); and dynamic qualities (force and timing) (Moore & Yamamoto, 2012). Although Sheets-Johnstone's movement descriptions are similar to Laban's elements, Laban’s framework provides more detail and is more widely used in movement analysis. Thus, this paper analyzes students’ movements using Laban’s movement elements.

The mathematical task

Nic watches a game where a ball is being thrown around a group of people in a clockwise direction. The number of people in the group is called the *people number*. Each time the ball is thrown in a game it is thrown in equal size *place jumps*. Each person throws the ball to the person on their left the same number of *place jumps* away. When the ball gets back to the first person the game ends.



In some games (like the 5-people 1-place jump game and the 5-people 2-place jump game) Nic notices that all the people throw the ball. In other games (like the 4- people 2-place jump game) only some people throw the ball. Nic wonders whether everyone gets to throw the ball in a 4-people 3-place jump game and a 6-people 3- place jump game.

Nic wants to make a dance using this game with people moving between each of the positions instead of the ball being thrown. Nic wants to know if everyone gets to move for different size *people number* and *place jumps*. Create and explain a shortcut that Nic could use for any size of *people number* and *place jump* size. Present this shortcut in the last 5 minutes of the session.

Figure 1: The task

As a modular arithmetic task, the modulus, n, is the people number and the place jump number is repeated addition (multiples of), m. For example, in the five-person three-place-jump game: the multiples of 3 mod 5 are (3, 1, 4, 2, 0/5). As this is the set of numbers in modulus 5, everyone gets to throw the ball (or move/swap places in the dance). A possible shortcut could be written as: if the people number and place jump number don’t share a common factor everyone gets to throw the ball (move/swap places).

The setting

This paper is part of a larger thesis studying students’ movement, as they explore a mathematical task and follows on from the analysis of students’ mathematical problematizing reported in Gandell and Maheux (2019). For Maheux & Proulx (2015) problematizing is the posing and solving of smaller self-generated problems in response to a mathematical task. The participants, four non-maths major students aged 18 and 22 years, were recruited from a six month tertiary bridging programme which provides entry into degree and diploma programmes.

During an hour-long session, the students engaged with a mathematical task in an open room with no tables and chairs. The task (Figure 1), printed on A3 paper, was attached to one of several vertical whiteboards positioned at the edges of the room with whiteboard markers and magnetic counters. All student activity was captured by three video cameras positioned at the edges of the room. The fragment transcribed below, occurs thirteen and a half minutes from the beginning of the session, and six and a half minutes after the first fragment transcribed in Gandell & Maheux (2019). The session began with a movement warm up, led by the researcher, who then invited the students to move freely around the room. The students then alternately used the vertical whiteboard and task sheet, and acted out two games from the task (a four-person three-place-jump game and a six-person three-place-jump game) in the open area of the room. In the first enactment the students used a counter as a ball, in the second enactment the students changed the game into a dance, as requested by the task (Figure 1).

Mathematical thinking in movement

At the beginning of this fragment the students have just completed acting out a six-person, three-place-jump game, which they call a six-three. As 3 modulus 6 has only two multiples (3, 0/6) only two people move, during the enactment of the game, which the students verbalize as swapping places. Returning from the open area of the room to the white board and task sheet they utter "that one doesn't work". The group, Chas, Kit, Ala and Paige, stand quietly for a few minutes. Kit first problematizes finding a formula, and then how to change a six-three game so that everyone can move.

1 Kit: A six-two for everyone to move I think (suddenly dabs three discrete positions with this right index finger, inscribing a circular path in front and to his right side with this right arm, and gazing first between Chas and Ala then towards Ala and Paige, Figures 2 a, b and c).

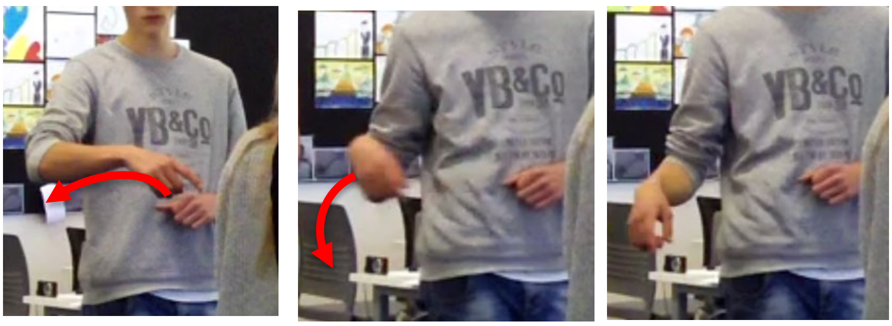


Figure 2: a) Kit dabs position one b) position two c) and position three

For Sheets-Johnstone (2011) "thinking in movement is not that the flow of thought is kinetic, but that the thought itself is" (p. 421). Thinking in movement does not require students to learn which movements will produce specified answers, or to pre-plan movements to express ideas: thinking in movement is the spontaneous activity of a dynamic thinking body. In the fragment above Kit demonstrates spontaneous, self-generated, mathematical thinking in movement providing a mathematical solution for his problematization of how many people move in a six-person two-place jump game (line 1).

In Line 2 Kit verbalizes that everyone will move, for a six-person two-place-jump game and performs a movement indicating three positions in a rough circle with his right index finger (Figure 2). Kit performs this movement very quickly, with the three positions distinguishable by the variations in dynamic qualities. The curved paths between each of the positions have a light, gliding quality, like a bounce. Kit pauses, at the end of each bounce, with a heavier and more bound quality, like a dab. Although these bounces and dabs serve to indicate different three positions to his right front and side (Figure 2), the sudden and indirect qualities of this movement give Kit's performance the feeling of a sketch. With his movement in line 2 Kit appears to be trying out, rather than defining, the solution of three positions for the game.

Kit clearly provides a solution for the six-person two-place-jump game with his movement: a solution not available from his verbalisation that everyone moves. No previous movements in this session have sketched a solution using Kit's bounce and dab movement, so Kit is not reproducing a movement. Thus, a new mathematical movement has emerged spontaneously from a moving dynamic body. As Kit's movement (Figure 2) does not replicate his verbalized solution the movement cannot be a pre-planned or pre-thought embodiment of that verbalization. In line 2, then, Kit performs a mathematical solution in movement: Kit is thinking in movement as Sheets-Johnstone (2011) describes.

Evolving thinking in movement

2 Chas: The trouble is instead of swapping (elbows bent index fingers touching, right finger traces horizontal curve forwards, left hand traces straight line backwards and up, Figure 3a) you go around (spiral trace with right arm across left, up and forwards, Figure 3b).

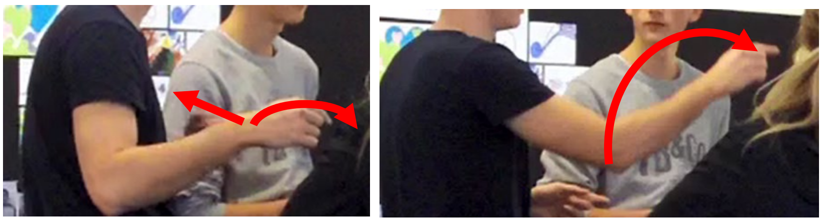


Figure 3: a) "instead of swapping" b) "you go around"

3 Paige: (lifts hands to hips) yeah that's what I was thinking

4 Chas: (touches left and right index fingers in midline) it goes around to the next person (spiral right arm trace left, forward and up across his body) instead of swapping (index fingers touching, traces two lines right hand forward, left hand backwards and up then rolls right hand backwards under left hand going forwards, then rolls right hand forwards under left hand going back, Figure 4a). So that's the swap (Rotates right and left hands around each other tracing horizontal circles Figure 4b. Holds final position index fingers pointed upwards for 2 seconds, Figure 4c).

Figure 4: a) over and under rolls b) spiral around vertically c) final held position

5	Chas:	From there (points right arm to left side, rotating torso and head to left, Figure 5a) goes around to (traces horizontal circle around body with right arm, rotating torso, to point to right side. Holds arm extended to right and looks back to Kit, Figure 5b) … the next person

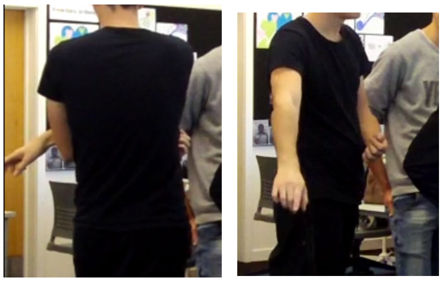


Figure 5: a) "from there" b) "to the next person"

Thinking in movement arises from a body that resonates with the world (Ingold, 2013; Sheets-Johnstone, 2011). Rather than reified minds creating symbols and representations of the world, new things, including mathematical things, emerge as tactile-kinaesthetic bodies correspond with animate material flows. Thus, thinking in movement is emergent, evolving and adapting to an ever-changing environment, and may take many different forms (Sheets-Johnstone). Chas, in lines 2, 4, and 5, demonstrates an evolving thinking in movement which contrasts with his almost unchanging verbalizations.

In line 2, Chas verbalizes and performs two problematizations which he differentiates both verbally and by using different space, body and dynamic movement qualities. For the first problematization Chas traces a line with each hand (Figure 3a) as he utters "instead of swapping back". In the second problematization Chas performs a spiral trace with his right arm (Figure 3b) uttering "you go around". The small, direct, straight line "swap" movements are performed with two hands, tracing a mainly horizontal pathway. In contrast, for the larger spiral "around" movement, Chas's right arm traces a path to the left, across his midline, then up and forward, through horizontal, vertical and transverse planes, with a sustained, indirect, light, floating, quality. Although Chas uses different words for his verbalizations, how "swap" might be different to "around" is not clear without his movements. Chas verbalizations, then, appear to be labels supporting his movement problematizations.

In line 4, Chas begins by repeating the swap and around problematizations, with little change to his movement or verbalizations, but in reverse order. Immediately after he repeats the 'swap' movement from line 2, Chas begins a new movement (Figure 4a) rolling his hands over and under each other in vertical semi-circles. Chas then verbalizes "so that's the swap", as he spirals his hands around each other in a horizontal plane (Figure 4b), finally stopping and holding the position in Figure 4c. Thus, in line 4 Chas performs three distinct "swap" movements.

These "swap" movements, which are performed in quick succession, are differentiated by transforming dynamic qualities from sharp, bound and direct to more continuous, freer and indirect. In this way the "swap" movement appears to enfold the dynamic qualities of both the "swap" and "around" movements, initially performed in line 2. In addition, the movement pathways change from almost straight lines to semi-circles and finally to spirals. Although performed with two hands the spiral pathway of this final swap movement (Figure 3b) reflects the spiral of the initial "around' movement from line 2. However, while Chas performs ever-evolving “swap” movements in line 4, he continues to verbalize these movements as a swap.

Finally, in line 5, Chas places himself in the centre of the movement and performs a large continuous, sustained horizontal curved pathway with his right arm. By combining a curved path, a sustained floating dynamic quality, and the use of one arm this final movement performs some elements of the “around” movement from line 2. However, in this movement Chas also includes elements of the line 2 swap movement, performing a more horizontal, single direction, pathway and a more direct dynamic quality. Thus, Chas’s movements (lines 2, 4 and 5) evolve and, by merging elements of both the “swap’ and “around” movements, seem to resolve the problematization of how counting around becomes swapping places in the game as a dance. Although Chas’s movements transform, in lines 2, 4 and 5, his verbalizations remain very similar, with his final verbalization indicating positions rather than referencing any resolution to his problematizations. By adapting and evolving his movements to merge his two problematizations, Chas shows how thinking in movement may evolve and change as a dynamic body resonates with an ever-changing environment.

Discussion and conclusion

For Sheets-Johnstone (2011) humans move and know the world through their tactile kinaesthetic bodies. Humans cannot remove themselves from their bodies and think in some disembodied mind. As this paper demonstrates, students’ movement is not only integral to their thinking and knowing, students think in movement. Kit’s movement illustrates how students animate bodies perform mathematical thinking: thinking that may not necessarily be articulated, expressed or made available in any other way except in movement. Similarly, Chas demonstrates how thinking in movement can emerge and evolve, even while accompanying verbalizations remain static. Thinking in movement is not about making decisions about how to move or where to move, rather spontaneous movements emerge and evolve in correspondence with an animate world (Ingold, 2013; Sheets-Johnstone).

For a long time, western mathematics education has de-privileged movement and the body in mathematical thinking. By ignoring students’ spontaneous thinking in movement, mathematics educators and researchers may be missing valuable instances of students’ mathematical thinking and knowing. As Sheets-Johnstone (2011) explains

> “thinking in movement is a way of being in the world, of wondering or exploring the world directly, taking it up moment by moment and living it in movement, kinetically. Thinking in movement is clearly not the work of a symbol making body, a body that mediates its way about the world by language, for example, it is the work of an existentially resonant body” (p. 425).

Students’ spontaneous movements provide access to mathematical thinking and knowing that may not otherwise be available. To fully understand students’ mathematical thinking, mathematics educators need to develop approaches that recognize and support students’ thinking in movement, rather than considering movement as an adjunct to verbalization or an expression of concepts held in a mind. In research, and in the classroom, mathematics educators need to provide not only space and support for students to move, but also need to pay closer attention to students’ thinking in their spontaneously performed movements.

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