

WORKING GROUP 12

From a study of teaching practices to issues in teacher education

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FROM A STUDY OF TEACHING PRACTICES TO ISSUES IN TEACHER EDUCATION

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ORGANISATION

Group 12 received 30 proposals. 4 were rejected, 3 were derived to other groups, 2 were accepted to present at the conference, but not for publication (only 1 author attended the conference), and 21 were accepted both for being presented at the conference and for being published in the proceedings (2 authors did not attend the conference). Therefore, the contribution of Group 12 for the proceedings consists of 19 whole papers and the summary of a paper. Each paper was reviewed by the leader, one of the co-leaders and 2 other authors.

39 researchers took part in the sessions, which were organised on the basis of 3 topics within the domain of teacher education and in relation to the title of the group. Panel I and IV were about *Understanding practice, understanding and promoting the mathematics teacher's development*, panel II was about *The process of becoming a mathematics teacher*, and panel III dealt with *Means, resources and methodology to research on and promote the mathematics teachers' development*. In each panel 5 authors presented briefly what their paper contributed on the topic of the panel. Next the whole group splitted into smaller groups, each one of the authors presenting their papers being in a different group. Before that everyone had received a sheet with the questions which were posed to the presenters' papers before the conference. The groups dealt with these and other questions and, in the following whole group session, posed, to the authors or to the whole group, several questions that emerged from the discussion within each group.

PANELS

It follows the summaries of the 4 panels.

- SUMMARY OF PANEL I: *Understanding practice, understanding and promoting the mathematics teacher's development, I*

It began with the short presentations by Janet Ainley, Iris Attorps, Alena Hospesová and Marie Tichá, Jean-baptiste Lagrange, and Alain Marchive.

Janet Ainley introduced the concept of **attention-based knowledge** as a kind of knowledge that enables experienced teachers to respond effectively to what happens during lessons.

Iris Attorps focused her intervention on dealing with **pedagogical content conceptions** of secondary teachers about the purposes of teaching Algebra from a **phenomenographic** perspective.

Alena Hospesová and Marie Tichá proposed **teacher's competence** as a broader term than teacher's knowledge, which includes the capacity to react to the situations that arise in the classroom and reflect on them.

Jean-Baptiste Lagrange approached the comparison of two models applied to the teachers' **use of technology**: one model addressing teachers' **views of successful use**, and the other one focusing on the **teaching practice** at the classroom.

Alain Marchive dealt with the **function of ritual practices** in the mathematics lessons in relation to the definition of teaching situations and to the structuring of pupils' actions.

The work of the groups after the presentations lead us to reconsider language: **what do we mean by** intuition, competence, conception...and even knowledge (sometimes taken for granted, but usually problematic). Another issue that arose was whether the papers are related to classroom teaching in the sense **whether they contribute to the way of thinking about classroom**. The issue of how do we **move student teachers from novice to expert state** was taken into account in the discussion, including the need of identifying such states. This issue is related to that of **how to develop teachers' knowledge or competence** in in-service education, and the conceptualisation of that **knowledge as a process**.

- SUMMARY OF PANEL II: *The process of becoming a mathematics teacher*

The panel began with short presentations by Laurinda Brown, Joao Pedro da Ponte, Ewa Swoboda, Pedro Gómez, and Stephanie Prestage and Pat Perks.

Laurinda Brown described her way of working with student teachers. Issues are identified from their experiences and practices in the classroom. These become, for them, motivations or **purposes for development**.

Joao Pedro da Ponte described different modes of **virtual communication** with student teachers on a *practicum*. These included a more public 'Forum' board and more personal email. For all students it required some time and effort to communicate their ideas to the Forum.

Ewa Swoboda examined the **attitudes** of elementary school teachers and student teachers to school mathematics. Her main finding was the contradictory nature of their opinions; she suggested that this could produce some dissonance in their classrooms.

Pedro Gómez examined whether preservice secondary teachers can be established as a **community of practice** by working in groups on the didactics of specific topics. He concluded that this is possible, though sometimes the knowledge constructed in these small communities is problematic by conventional norms.

Stephanie Prestage and Pat Perks explained how *tools* for learning can become *rules* for social regulation, or be perceived as such. They exemplified this dilemma with reference to the use of '**learning objectives**', but their work has wide-ranging implications.

The work of the groups after the presentations has led us to consider issues such as:

What is a community of practice? How far can Wenger's ideas be pushed and still be faithful to his original meaning and intention?

To what extent the community of practice - originally an analytical tool - can be a prescription for practice. Analogies were made with 'constructivism'.

When does a community of practice become a community of inquiry? Or a community of enquirers.

What meanings do we attach to 'learning'? - distinguishing between descriptive ('did') and normative ('should').

Do some teachers (and/or student teachers) develop a 'reflective stance' early on? If so - why, how?

What is known about teachers' perceptions of teaching – a job or a vocation?

How does the role of the teacher educator differ in different media (internet etc).

What are the different dimensions and continua within which we can conceptualise and think about teacher development?

- SUMMARY OF PANEL III: *Means, resources and methodology to research on and promote the mathematics teachers' development*

It began with short presentations by Pilar Azcárate, Liz Bills, Lalina Coulange, Bodil Kleve and Leonor Santos.

Pilar Azcárate presented her study about the role of **learning portfolio** as a tool for teacher development in pre-service secondary teachers education.

Liz Bills focused her intervention in a reflection on the use of **models** to deal with complexity and their relationship with values.

Lalina Coulange approached the case-study of a teacher conducting an ordinary lesson of algebra, using both the theory of **didactic situations** (Brousseau) and the **anthropological approach** (Chevallard).

Bodil Kleve analysed teachers' implementation of a curriculum reform in Norway. Her study shows that the **relationship between beliefs and practice** is not straightforward.

Leonor Santos focused her intervention on production of a **portfolio** in the subject of Didactics of Mathematics of pre-service mathematics teachers education for secondary school. She referred the potential of a portfolio as well as some difficulties students encounter, as well as its potential for teacher educators reflect about her own practice.

After the presentations, the large group splitted into four small groups. One of the themes developed was related to Kleve's paper – the consistency between beliefs and practice observed. One question raised was: which teacher development may help to reflect on the way he/she teaches in the classroom? Portfolios seemed to reduce the gap between teachers' beliefs and teachers' practice.

Another issue related to classroom is about learning while teaching and learning by reflecting after lesson. The question raised was: what kind of tools teachers should have to learn while teaching?

The issue of values was stressed as well as the danger and difficulties to separate values from other aspects of teaching. It was also mentioned that when we are speaking about values we are addressing the all teaching and this is different when we refer to primary or secondary teachers.

About portfolio was stated that portfolio is just one strategy and not the strategy. It was to have attention as to use the same tool as a reflection tool and an assessment tool. Portfolio may be an instrument to promote teacher reflection and when used in pre-service education may be an instrument for teacher educators reflect on their own practice.

- SUMMARY OF PANEL IV: *Understanding practice, understanding and promoting the mathematics teacher's development, II*

The panel began with short presentations by M^a Cinta Muñoz, Susana Murillo, Tim Rowland, Gerard Sensevy and Jeppe Skott.

Muñoz reported on a case study of an elementary school teacher following her at the transition stage from initial training to the immersion into practice. The researchers studied the teacher's professional development, examining the nature of her reflection as a student teacher on the practice of another teacher and her reflection on her own practice when she became a teacher. They also studied the influence of the teacher's previous experiences on her professional development.

Murillo presented a French view on high school teachers' ways of handling students' errors related to the notion of inverse function. Her theoretical framework was based

on French theoreticians. The analysis focused on the teachers' written speech, classifying speech role with relation to students' error types.

Rowland described a framework for the identification and discussion of prospective elementary school teachers' mathematics content knowledge as evidenced in their teaching. This framework - 'the knowledge quartet' - emerged from intensive scrutiny of 24 videotaped lessons. Application of the 'quartet' in lesson observation was illustrated.

Sensevy contrasted two epistemological positions of teachers and researchers, by means of their respective actions in a research process. The epistemological gap found between the teachers' stance and the researchers' was explained by the teachers' practices and the researchers' expectations and interpretations. Sensevy claimed that researchers need to understand the different constraints with which teachers need to cope.

Finally, Skott reflected on his experiences from a development programme for teacher education in Eritrea, emphasizing the need to consider practice when theorising practice. He claimed that the relationships between theory and practice can be described as a theoretical loop, starting from and returning to practice.

After the presentations the large group splitted into five smaller groups, each small group began work by discussing issues related to one paper with the author(s) of this paper, then extracting some key ideas and questions. The whole group discussion that followed the work of the smaller groups included short reports of the small groups work.

EMERGING ISSUES

In the discussions in the whole group and in the smaller groups some issues arose, which the participants considered relevant for the present and future work of the group. We can support some thoughts on them, but they remain open for us, and, in this way, they challenge us:

1. One of the issues that arose in the group was the demand for theories, perspectives and methods to contribute to the way of thinking about the classroom. It means that they should try to capture or approach the flavour and essence of the classroom activity.
2. Another related issue was the incompleteness of current models to give an account of the real teaching-learning process. Some attempts, like the consideration of Ainley's and Luntley's attention-based knowledge, or Bills' reflection on the use of models to deal with complexity, were discussed.
3. The relationship between methodological decisions and paradigms was also taken into account. In this concern we discussed the relationship between the researchers and the teachers (researchers as external observers and collaborative environments came into the discussion).

4. The dynamic characterisation of concepts like knowledge, pedagogical content knowledge and teacher's competence challenged traditional and national definitions of them. At the same time, the inclusion of values, issues related to communication amongst teachers in a (let's say) community of practice or the perspective of the socio-cultural theory helps us to make our approaches and understandings of teachers' professional development broader and more complex (in other words, helps us to understand better the teacher's practice).
5. With respect to the notion of community of practice (and the related notion of community of inquiry), we had several discussions about its application in teacher education. One reflection was we have to be careful because this notion does not come from the educational research and can become the rhetoric of our discussions.
6. In using ICT, we recognised that the role of the teacher (for instance in electronic communication) could be different from that in a classroom. The issues of the application of ICT to learning (eg e-learning) should be reflected on issues of the application of ICT to teaching (eg e-teaching). It was addressed specially the issue of communication and collaboration in teacher education and the classroom use of technology.
7. We examined the learning portfolio as an assessment instrument as well as a means to promote mathematics teacher development through reflection. One idea was that students must understand its purpose as a tool to support learning. But there may be some difficulties when the same portfolio works as an assessment tool and as a reflection tool. In pre-service teacher education it could be also a way to question teacher educator's own practice.
8. Finally, we valued the possibility to contrast several theoretical frameworks and discuss a couple of notions (tool, model, knowledge, learning...) a lot. In particular, we propose to go deeper in the confrontation of frameworks and models and to foster collaboration by a common work of analysis of some corpus of classroom teacher practice observation (in such a way one could organise some sessions in next conferences). This proposition would provide an opportunity to build from the experience of this group because a number of frameworks and models were proposed but it is not clear what each addresses and is relevant and efficient for.

The focus on teacher knowledge in Ainley's and Rowland's papers, or the development of teachers' competences in Hospesová's paper, or the relationship between institutional teaching practices and teacher's knowledge in Coulange's paper, the importance of conceptions in Attorps' and Swoboda's papers, of values in Bills', purposes in Brown's, the application of portfolios in teacher education in Azcárate's and Santos' papers, the constitution of communities of practice in pre-service secondary education in Gómez's paper, the ethnographic perspective of the implementation of a curriculum reform by Kleve, the complexity of the use of

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technology by teachers in Lagrange's paper, the concern with the role of rituals in mathematics lessons by Marchive, the focus on the transition from initial training to the immersion in practice by Muñoz, the treatment of students' errors in Murillo's paper, the concern on virtual interaction in pre-service teacher education in Ponte's paper, the importance of learning objectives within a socio-cultural activity system in Prestage's paper, the relationship between teachers and researchers by Sensevy's, the theoretical loops of Skott to deal with the relationship between theory and practice, give an overall, colourful picture of the work and interests of the researchers participating in this group.

WHAT TEACHERS KNOW: THE KNOWLEDGE BASES OF CLASSROOM PRACTICE

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Abstract: *We report on a pilot project that has investigated the hypothesis that, in addition to subject and pedagogical knowledge, much of what experienced teachers know is what we call attention-dependent knowledge, and that it is this knowledge that enables them to respond effectively to what happens during lessons. A study of mathematics lessons taught by six teachers has led to some further conjectures about the role of attention-dependent knowledge in teaching, and about the interplay between different knowledge sources in planning and teaching.*

Keywords: teacher knowledge, attention, expert practice.

In this study¹ we have attempted to address the question of how experienced teachers deal with the enormous complexity of classroom environments and what it is that teachers can learn from the experience of teaching particular classes which enables them to apply their expertise to teaching other very different, classes.

We argue that attempts to describe the knowledge base of teachers in terms of subject knowledge and general and subject-specific pedagogical knowledge (e.g. Shulman, 1987) may offer tools for analysing particular aspects of practice, but fail to provide an adequate account of what is required to function effectively minute by minute in the classroom. There have been a number of studies which have attempted to give accounts for the ways in which teachers make choices about how to act ‘in the moment’, for example, in terms of decision trees (Peterson and Clark, 1978), or the balance of influence of knowledge, beliefs and goals (Schoenfeld, 1998).

In contrast to these relatively complex accounts we offer a different hypothesis: much of what experienced teachers know is what we call *attention-dependent knowledge*. This attention-dependent knowledge not only is not reflected in what is written down in lesson plans, but *cannot* be written down. However, we conjecture that it is this knowledge that enables teachers to respond effectively to what happens during the lesson. Understanding the performance of experienced² teachers requires an account of the interplay between the subject and pedagogic knowledge that will be articulated in learning objectives and lesson plans, and attention-dependent knowledge that can only be revealed in the classroom. It is well documented that experienced teachers

¹ ‘Attention and the knowledge bases of expert practice’, funded by an AHRB Innovation Award. A report of this project can be found at www.warwick.ac.uk/fac/soc/philosophy/research/akbep

² By ‘experienced’ we mean those who have developed their expertise through experience; this is not the same as simply counting years in the classroom.

often find it difficult to articulate what it is that they do successfully in the classroom, other than in highly situated accounts of particular pupils or aspects of the curriculum (e.g. Edwards & Collison, 1995). We argue that it is attention-dependent knowledge, and the skills which give access to this, which teachers find difficult to describe, possibly because of the relative lack of attention paid to such learning in formal teacher education (Edwards & Protheroe, 2003).

In this study we have developed a methodology for a qualitative exploration of this hypothesis by looking for evidence of the existence of attention-dependent knowledge, characterising attention-dependent knowledge and locating its role in shaping teacher deliberation in class and legitimising expert performance.

Theoretical framework

The study offers a novel conceptual apparatus for understanding the role of attention-dependent knowledge. The conceptual innovation is to characterise the situatedness of attention-dependent knowledge in terms of specialised attentional skills. The idea is that experienced teachers have a repertoire of attentional skills for attending to cognitive and affective aspects of pupil activity. In other words, experienced teachers 'see' the classroom situation differently from novices. Similarly, Edwards and Protheroe (2003) claim that student teachers are more likely to 'close down on complexity' in the classroom.

The teacher's attentional skills are generalisable. The knowledge they make available on any given occasion is, however, highly situated and is often only expressible in a contextualised proposition as a response to 'that situation', 'this cognitive difficulty/insight, etc.' A teacher's response to a situation, characterised in this way, is highly particular and not a response driven by a general rule that could have been articulated in advance of the teaching encounter. Edwards and Protheroe (2003) argue that current approaches to initial teacher education in the UK are underpinned by a model of professional knowledge as something which can be 'called up and applied' and offer a critique of initial teacher education which does not offer opportunities to develop what we would call 'attentional skills' through peripheral participation in the practice of experienced teachers.

Furthermore, attention is an active perceiving and involves selection on behalf of the subject. The knowledge which is gained by and from this attention informs subsequent actions. This means that the concept of judgement, rather than rule-following, lies at the heart of the account we offer. Fuller theoretical discussion can be found in Luntley (2004).

The empirical study: developing a methodology

In this exploratory study, our initial approach was to watch some lessons, identify the 'episodes' in which we felt that teachers were acting on the basis of attention-dependent knowledge, and then interview teachers about them. The remaining sections discuss the development of our methodology and present some initial

findings. We worked broadly using a grounded theory approach. Although data collection and analysis are described separately here, they were largely interwoven.

The study was carried out with an ‘opportunistic’ sample of experienced teachers, two in a primary school, and 4 teaching mathematics in secondary schools. There were two cycles of observations. In each cycle one mathematics lesson (and occasionally two) from each teacher was observed, and recorded using a video camera and a radio microphone. The lessons to be observed were chosen by the teachers, generally on the basis of convenience. We did not ask the teachers to give us written lesson plans, as this would have imposed a level of formality which we wanted to avoid. However, whenever possible we had a brief discussion with the teacher about their plans immediately before the lesson.

Three members of the research team were present in each lesson, one operating the camera, and the other two making unstructured observation notes. The video camera was focussed on the teacher throughout the lesson. The audio tape was transcribed in full straight after the lesson. Later the transcripts were annotated to add non-verbal behaviour and contextual detail from the video tape. The aim of the observers in the lesson was to identify episodes in which the teacher appeared to be acting on the basis of attention to aspects of the classroom activity, rather than in ways which could have been predicted from a lesson plan. Clearly there could be very many instances of such behaviour in any lesson, since even the most detailed lesson plan will not specify the exact words to be spoken, or the pace and nuances of speech. Our observations needed to focus on incidents that were accessible to observers as the lesson progressed. Typical examples of potentially interesting episodes were when a pupil was unable to answer a teacher’s question or gave an answer which was clearly unexpected, when a pupil asked for help, or was clearly confused or inattentive, when a teacher appeared to change the pace or direction of the lesson.

After the lesson, the researchers exchanged initial impressions about their observations. Two days later, they met to discuss the lesson in more detail, with both the video tape, and the transcript available. They used their notes to identify the episodes in the lesson that would form the focus of an informal interview with the teacher, which took place immediately after this discussion. This was structured around watching the video sequences. Interviews were audio taped, and full transcriptions made. The transcriptions of the lessons and the related interviews were subsequently coded using categories which emerged during the data analysis.

Developing the approach to data collection

The key features of our methodology were the researchers’ ability to identify the kinds of episodes that we were interested in exploring as the lesson was in progress, and developing an interview strategy that would enable the teachers to talk about their actions during those episodes. Identifying potentially interesting episodes involved speculation about what had prompted a particular action. We were creating stories about what we had observed, and inevitably our stories were a function of our

attention during the lesson. Initially, there were some interesting differences in the ways in which each of us attended to the progress of the lesson, which we might attribute to our differing professional backgrounds. Re-viewing parts of the lesson through the video recording was therefore important in our identification of episodes. As our experience of the individual styles of the teachers increased, and we developed a clearer picture of the kinds of episodes which were proving interesting, there was an increasing level of agreement in the examples identified.

During the interview, one of our concerns was to test out our stories about the episodes. In some cases these turned out to be mistaken: what we took to be a spontaneous decision had actually been planned, or the interaction with a particular child was based on previous history. In other cases, the teacher did not have particularly clear recall of the episode, even having seen it again on the video. In order to maintain a neutral approach, the technique we adopted was for one researcher to provide the basic structure of the interview, setting the scene for each video extract, and using an opening question such as ‘What’s going on here?’. The second researcher then brought in different questions to try to probe the teacher’s thinking further. Productive questions which emerged were:

- If you could run that lesson again, would you change anything?
- Can you think of a similar occasion when you have acted in the same way/differently?
- Is that a common strategy for you to use?

Without exception, the interviews were relaxed occasions. As they became more familiar with our style, the teachers often offered spontaneous comments in response to the video extracts. All the teachers seemed to enjoy the opportunity to discuss their pupils and the content of their lessons in this way.

Analysing the data

Our first attempts at analysing the data focussed on the episodes themselves, based on the lesson transcripts, the observation notes and the video recordings. We coded contextual details (e.g. whether the episode involved an individual, group or the whole class, whether it was initiated by teacher or pupils), and the underlying focus of the episode, as **cognitive** or **behavioural/affective**. The vast majority of the episodes we identified were cognitive, and we sub-divided these into **cognitive problems**, where pupils were showing differing understandings of mathematical ideas and the teacher was trying to address this, or **cognitive opportunities**, where the teacher was trying to extend the pupils’ thinking.

A second kind of coding of the episodes was to distinguish between occasions when the teacher seemed to be **reacting** to the classroom context by using a familiar strategy, and those when the teacher was **responding** in a novel way (Mason, 2002). The distinction between reactions and responses was not, however, always clear-cut. A further coding was to indicate whether, at interview, the teacher seemed to have

been aware of making a (**conceptual**) choice in that particular episode, or whether their reaction/response had been more intuitive (**non-conceptual**). This distinction was also not always straightforward.

The interview transcripts were initially coded in fairly pragmatic ways. More detailed analysis of the interview transcripts provided some clear evidence that teachers were acting, in part, on the basis of attention-dependent knowledge. Their accounts contained references to (for example) the expression on a particular child's face, a sense of restlessness in the class as a whole, an interaction they had observed between particular children. Further, it emerged that for many of the episodes the teacher talked explicitly about what they thought underlay particular actions on the part of the pupils. Our most recent analysis of the interview data has identified sections of the teachers' accounts that indicated that their attention had been focused on what pupils' were attending to. This knowledge about the pupils' attention seemed to be particularly significant in episodes in which we saw teachers moving the mathematical content of the lesson forward.

The significance of this form of attention did not emerge until after the data collection was completed. For some episodes, the interview transcript provides evidence for the teachers' attention focussing on what pupils are attending to. For others, there is nothing explicit in the transcript. This may be because the teacher was not, in fact, attending in this way, or it may be that the structure of the interview did not support discussion of this. It would be a priority in future research to adapt our interview technique to try to elicit such commentary, for example by asking 'Did you have a sense of what the pupils were thinking about?'

Further analysis of episodes in which the teacher is attending to pupils' attention led to a further coding. In some the teacher appears to be **interrogating** the mismatch between the pupil's attention and the teacher's expectations in a way that allows the teacher to adapt their teaching to move towards shared attention. In others, although the teacher is clearly **noting** the pupils' attention as different from their expectation, they do not work directly on this difference, but use some other strategy.

Planning and teaching styles: exploring the interplay between knowledge bases

Within the group of six teachers in our study we observed a range of styles in the way in which teachers both planned their lessons and worked with these plans in their teaching, which suggested that teachers were drawing on different knowledge bases (subject knowledge, pedagogical knowledge and attention-dependent knowledge) in different ways. The three accounts which follow are not attempts to characterise individual teachers. Rather they are sketches drawn from our data of particular episodes, amalgamated to characterise distinctive styles observed.

Clinging to the lesson plan

Jenny knows her primary-school class extremely well. The episodes that we identified in her lessons suggest that she attends closely to patterns of behaviour which give her insights into both cognitive and affective issues.

In one lesson Jenny asked Colin a question that he was unable to answer. After looking at him for a few moments she said 'Not sure? Don't worry' and then asked another child (Hilda) to give the answer. Later Jenny returned quietly to Colin and asked if he had understood Hilda's explanation. After Jenny had watched the video of this episode we asked her what had made this approach feel right.

I could sense a sort of panic in Colin that I didn't want to make worse. And yet I banked on Hilda knowing it. ... So I could reinforce it for everybody at that point and then I could go back to Colin and ease that worry that he was having. That panic that he was feeling. He won't say that he is struggling ... but you can see it in him. There is this sort of rising panic.

We categorised this episode as **Cognitive problem/Affective, Reaction, Non-conceptual, Noting**. Jenny **reacted** to Colin's 'rising panic' intuitively by using a familiar strategy of asking another child to help out. She was able to **note** Colin's difficulties, but her priority was to keep the lesson moving for the whole class.

Later Alan offered an explanation which showed a more sophisticated level of reasoning than Jenny had expected. During the lesson, Jenny appeared to challenge his reasoning, but in the interview she commented:

I couldn't work [out] where I was going next, what I was going to do. I knew Alan was right and I thrilled to bits that he made the link ... he was being logical and it was a great piece of working definitely. I just couldn't get my head around why I was going to do it and what I wanted to do next. ... I was actually completely thrown by it

We categorised this as **Cognitive opportunity, Reaction, Conceptual, Noting**. Jenny **noted** Alan's idea, but did not develop it, and moved on. Later she said:

So if I hadn't been following the script I would have done [another activity] because it was just perfect wasn't it? And it was the perfect opportunity but because I needed to get on, ... I didn't and I should have and I knew it at the time and I was debating whether I should just go with it but I knew I'd run out of time and I knew I wouldn't get anything done that I wanted to do ... I did want to make sure I could get through everything and make sure that they understood.

In a lesson on percentages children had become confused when they met a problem about percentages of 200. In the interview she acknowledged that she knew that most of the class were confused, but went on:

One of those times when you think, you know, Oh my God! What do you do next? But if I would have thought, which I didn't today, cause I was in a panic with you there ... If I'd have thought of using the numberlines then, with the percentages at the bottom, they would have seen instantly why it wasn't 138%, and we could have worked it out from there. But it was sheer, utter and total Oh my God moment.

We categorise this episode as **Cognitive problem, Reaction, Conceptual, Noting**.

Jenny clearly has the attentional skills which give her access to attention-dependent knowledge about her pupils' understanding, or lack of understanding. She plans her lesson in great detail, focussing on specific learning outcomes, and often makes use of planning resources provided as part of the National Numeracy Strategy. However, she is not a mathematics specialist, and we see here weakness in her subject knowledge which makes her feel she has to 'stick to the script', overriding the attention-dependent knowledge she gains during the lesson. In episodes in Jenny's lessons we see more **reactions** than **responses**, and although she often **notes** pupils' attention, there are very few instances of her **interrogating** this to develop thinking.

Going with the flow

Alice is a secondary mathematics specialist who appears very confident in her subject knowledge. She prepares her lessons thoughtfully in terms of tasks and resources, but her planning relies less than Jenny's on detailed learning outcomes.

In a lesson on quadrilaterals, the class played a matching game with shapes, which included both quadrilaterals and triangles, and Alice realised that they were not as familiar with the vocabulary of shape names as she had anticipated. Alice collected a list on the whiteboard of the shape names that the pupils said were difficult (Parallelogram, Isosceles right angle triangle, Scalene triangle, Rhombus, Quadrilateral, Arrowhead, Isosceles trapezium). Alice wanted to focus on quadrilaterals, so her first strategy was to eliminate the two triangles from this list. She asked, "Can you work out which of those two words don't fit with the rest?". Tod responded, "Rhombus and arrowhead?" but was unable to offer a clear explanation of why he thought this. Alice asked another pupil, who replied, "Is it rhombus and arrowhead because they're not like - they're not like a certain shape."

After getting one or two more responses which did not identify the triangle names, Alice changed approach and focused on each item in the list in turn, asking pupils to describe and draw it. At interview Alice made the following comment:

I had no idea what it was that [Tod] was trying to say. I couldn't see any link between the two he had given me. I couldn't think, arrowhead and rhombus? What are the ... Apart from the fact that the words themselves may be as opposed to the shape. And I had no idea. And when the next person said the same two things, I was beginning to think: Oh God! There is something I am missing here. [Laughter] Something that is obvious to them but not obvious to me. Because you know sometimes with child's eyes you see something. Then I realised that they obviously didn't even look at those words and think, oh that's a three sided, that's a four sided. They obviously didn't have that connection as an obvious connection between the number of sides and the actual words. There was obviously something else they were looking at, if you know what I mean. Which is

why I then thought I am going to have to try and pull out here how many sides do these things have.

We categorised this episode as **Cognitive problem, Response, Conceptual, Interrogating**. Alice recognised a mismatch between the pupils' focus of attention and her own, and was able to **interrogate** this in order to **respond** in a way which changed the direction of the lesson, but enabled her to re-focus the pupils' ideas.

In another lesson pupils were practicing their skills with using compasses to draw perpendicular bisectors of line segments. Alice had set the exercise in the context of bisecting the sides of a triangle, hoping that some pupils would get as far as finding that the three bisectors cross at a single point (the circumcentre). After a demonstration on the whiteboard, pupils were asked to draw 'any triangle' in their exercise book, and then draw the perpendicular bisector of each side. While moving around the class, Alice noticed that several pupils had become confused with their drawings. She asked the class to stop, went back to the whiteboard, wiped off the original drawing of a triangle, and instead drew a single line. She then demonstrated the process of drawing the bisector again before adding a second side of the triangle, and indicating that the process had to be repeated.

Alice described what she thought the pupils had 'got in their heads'.

Alice: A lot of them were leaving it to two arcs and not cutting the line so they were going like that and like that and they thought they had done it. So they had lost sight of what the purpose was which was to cut the line in half.

Int: Ok and when you went you went to the board you didn't draw the triangle?

Alice: No because if I had drawn the triangle they would have got triangle in their heads instead of bisecting the line in their heads. I wanted to remind them that they were bisecting a line before reminding them that they were doing the triangle. Does it make sense?

We coded this episode as **Cognitive problem, Response, Conceptual, Interrogating**. After this episode, Alice again changed the focus of the lesson in response to attention-dependent knowledge about the pupils' progress. Later in the interview she commented specifically on her approach to planning.

we've got to write lesson plans and hand them [in] ... two weeks before you are going to be teaching some of those lessons and I can't do it. I've got colleagues who plan a whole term's lessons and try to stick to them but I tend to plan my lesson the night before really. I have in my head a long term plan, what I've got to do and I have actually written them down what I am going to do each lesson for the rest of the term, but that's just a single line and then you sort of construct your lesson around that. And that's just

to make sure that you are actually doing what's in the syllabus and get it covered by the end of the term.

Like Jenny, Alice appears to have good attentional skills which allow her to access attention-dependent knowledge about her pupils' focus of attention. However, her confident subject knowledge allows her to 'go with' what she learns during the lesson, and adapt her teaching accordingly. She does not feel the same need either to plan the structure and sequence of her lessons in detail, or to stick to the plans that she has made. The episodes in Alice's lessons show relatively more instances of **responses** and of **interrogation** of pupils' attention than Jenny's lessons.

Ploughing ahead

Like Jenny and Alice, Martha is an experienced teacher. She is a secondary mathematics specialist, and has sound subject and pedagogical knowledge. She plans her lessons around carefully chosen sequences of tasks. She is able to offer a clear rationale for her planning in terms of the difficulties her pupils experience in learning aspects of mathematics, and teaching strategies which she uses. However, observing Martha's lessons we were surprised to find many pupils disengaged from the activity, and relatively low levels of attention to Martha's presentation of the lesson. We found it difficult to identify episodes in Martha's lessons where we felt that she was drawing on attention-dependent knowledge, and of these there was only one instance where we felt that she was attending to the focus of pupils' attention.

A typical episode took place in a lesson on simplifying fractions. Kim had already offered 'four fifths' as a simplification of eight tenths. Damien then said (speaking rather indistinctly) 'is it two over two and a half?' This could have offered an interesting opportunity for developing the lesson. Martha, however, said '*That would be making it more complicated. That wouldn't be simpler, would it?*', and then continued. We conjectured that Martha did not want to risk confusion by exploring Damien's idea, but in the interview a different scenario emerged.

Martha: Now what did he say? Um, he was talking about one of the other fractions, I can't remember. I think was one of the fractions $2/3$ s? I think he said 22 over 33. Something like that.

Int: We think he said: 'could it be 2 over 2 and a half'.

Martha: I don't think he did. Now the reason why I say this is difficult is because I've had a similar class, well doing similar things, and someone, you know someone in another class did suggest something like that the other day. Something like 3.5 over something. But I don't think he did. No I can't actually remember.

Martha's account suggests that at the time she did not attend closely to what Damien was saying, and it is also somewhat unclear how well she recalled the incident. There

were several other episodes in which it seemed to us that Martha lacked awareness of things that were happening in the classroom which were apparent to us as observers. We conjecture that Martha lacked the attentional skills which would have allowed her to access attention-dependent knowledge, and that without this she was unable to put her subject and pedagogic knowledge into practice effectively.

Conclusions

On the basis of this small scale study, and the methodology we have developed, we have evidence for the existence of attention-dependent knowledge as part of what experienced teachers know, both in the sense that they have attentional skills which enable them to 'read' the activity of the classroom, and that they use the knowledge they gain by and from this attention in making judgements about how to act. Further, we argue that the recognition of attention-dependent knowledge is significant in explaining and justifying why experienced teachers act in the ways they do, and that the model of different knowledge bases enables us to give at least partial accounts of differing styles of planning and teaching. This may be seen as complementary to the more detailed model offered by Rowland et al. (2005).

On the basis of our study we also conjecture that as teachers develop their experience as successful practitioners, their use of attention-dependent knowledge, and particularly the ability to attend to and interrogate the focus of pupils' attention, will increase. We are currently planning an extended study that will allow us to explore this conjecture with novice and experienced teachers.

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SECONDARY SCHOOL TEACHERS' CONCEPTIONS ABOUT ALGEBRA TEACHING

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Abstract: *In this article secondary school teachers' conceptions concerning the purposes of algebra teaching are discussed. The data was gathered by interviews and videotapes. Both newly graduated and experienced teachers were participated in the study. The phenomenographic research method was applied in the investigation. The results indicated that the teachers present algebra as something to do rather than emphasising the central ideas and concepts of algebra. They comprehend algebra as a study of procedures for solving certain kinds of problems in everyday life and problem solving. The key instructions in this conception are simplify and solve.*

Keywords: algebra, conception, equation, mathematics teacher.

Introduction

Teaching of and achievements in mathematics have been criticised in several countries during the last decade. It is generally concluded that school mathematics focuses on developing algorithmic skills rather than mathematical understanding (e.g. Sierpiska 1994; Soro & Pehkonen 1998) and teachers devote much less time and attention on conceptual instead of procedural knowledge (Porter 1989; Menzel & Clarke 1998, 1999). Pupils learn superficially several basic concepts in arithmetic and algebra without understanding (e.g. Hiebert & Carpenter 1992; Sierpiska 1994).

In recent years, there has been a growing interest in mathematics teachers' conceptions about teaching and learning of mathematics. Teachers' mathematics-related beliefs and conceptions have been investigated in numerous research reports on the last decade (e.g. Adams & Hsu 1998; Pehkonen 1998). Also student teachers' conceptions of mathematics teaching have been studied (e.g. Trujillo & Hadfield 1999). There are still few studies concerning mathematics teachers' beliefs and conceptions of different content areas such as algebra. Most of the earlier studies are dedicated to teachers' beliefs and conceptions of mathematics, mathematics learning and mathematics teaching (Thompson 1992).

Interest in research about teacher knowledge has also arisen in recent years. Much current work on the development of a qualitative description of teacher knowledge and conceptions has been influenced by Shulman's model for teacher knowledge (Shulman 1986). Current research on mathematics teachers' subject matter knowledge, which includes knowledge of the content of a subject area as well as understanding of the structures of the subject matter (Shulman, 1986, 9), has been investigated in a large number of recent studies (e.g. Tirosh, Fischbein, Graeber & Wilson, 1999; Attorps 2003). The research results are essentially the same: teachers lack conceptual knowledge of many topics in the mathematics curriculum. Recent research on the relationship between teacher knowledge and teaching practice has

also pointed out the need to carry out more studies involving specific mathematical topics. Furthermore, the research has shown that the way teachers in mathematics instruct is determined partly by their pedagogical content knowledge i. e., knowledge that is specific to teaching particular subject matter (Shulman 1986, 9). In numerous research reports (e.g. Lloyd 1998) a strong interdependence of conceptions about subject-matter knowledge and pedagogical content knowledge has been documented. It appears that many teachers do not separate their conceptions about a subject specific topic from notions about how to teach that topic. Therefore teachers' subject matter knowledge influences their planning and classroom decisions (Brophy 1991).

In this study, the phenomenographic approach is used in order to reveal differences between the teachers' conceptions about algebra teaching. The approach illustrates in qualitatively different ways how a phenomenon is apprehended by individuals (Marton & Booth 1997). A person's knowledge of the world is regarded as a number of conceptions and relations between them.

In this paper I discuss one of the aspects of teachers' pedagogical content knowledge. The present study seeks to answer the following question: What pedagogical content conceptions do the secondary school teachers have of the purposes of algebra teaching? The study especially deals with the purposes of teaching the concept of equation.

Pedagogical content knowledge

Teachers' knowledge of teaching mathematics is based on their learning experiences in mathematics. This knowledge is developed during the studies of mathematics but most of this knowledge is acquired in teacher education, teacher practice or in the place of work (Ernest 1989, 18). Knowledge that is specifically connected with teaching particular subject matters is called pedagogical content knowledge (Shulman 1986, 9-10; Grossman 1990, 7; cf. Ernest 1989, 17-18). Pedagogical content knowledge is a term to describe *the ways of representing and formulating the subject that make it comprehensible to others* (Shulman 1986, 9). According to Brown and Borko (1992, 221) one of the most important purposes in teacher education is the acquisition of pedagogical content knowledge. In fact, it is recognised, that this knowledge forms the essential bridge between academic subject matter knowledge and the teaching of subject matter. It includes an understanding of which representations are most appropriate for an idea, which ideas are difficult and easy for learners, what conceptions and preconceptions that students in different ages hold about an idea. Particularly, if the preconceptions are erroneous conceptions, which they often are, teachers need to improve their knowledge of strategies in order to be successful in reorganising the understanding of learners (Shulman 1986, 9-10). Pedagogical content knowledge also includes conceptions and beliefs about the purposes for teaching a subject at different grade levels (Grossman 1990, 8; Ernest 1989, 20). A teacher in mathematics must have a clear conception of the purpose of teaching specific curricular topics such as algebra at school. According to Picciotto and Wah (1993, 42) the aim of school algebra should be an understanding of the

concepts where different mathematical tools and themes are considered to be vehicles and not the purpose of the course itself. Pupils need to absorb concepts such as functions, numbers, variables, operations, equations and mathematical structures; tools and themes may strengthen motivation (cf. The Swedish Board of Education 2000).

The curricula in compulsory schools in Sweden are designed to make clear what all the pupils should learn. ‘Goals to attain’ define the minimum knowledge to be attained by all the pupils by the end of the fifth and the ninth year at school (The Swedish Board of Education 2000). According to the curriculum in mathematics, the pupils should for example by the end of the fifth year be able to discover numerical patterns and determine unknown numbers in simple formulae in introductory algebra. Similarly, the pupils should by the end of the ninth year be able to interpret and use simple formulae and solve simple equations. The school in the teaching of mathematics, should aim to ensure that pupils develop their ability both to understand and use logical reasoning, draw conclusions and generalise, explain both orally and in writing and provide the arguments for reasoning. The goal of teaching of algebra should be that pupils develop their ability both to *understand* and to *use* basic algebraic concepts, expressions, formulae, equations and inequalities (The Swedish Board of Education 2000).

Method

Ten secondary school teachers in mathematics participated in the study. Five teachers were newly graduated (less than one year’s experience) and five were experienced (between 10 and 32 years’ experience). Data was gathered by interviews and videotapes. The interviews took place in the schools, where the teachers worked, and were recorded. Each interview lasted about two hours. Videotape recordings of six lessons, which the three (of five) newly graduated and the three (of five) experienced teachers had in algebra, gave further information about their purposes of algebra teaching at the school context. The interview quotations have been marked in the following way: For example I1 = Interview 1, p1 = page 1 in transcribed protocol, V1 = Videotape 1 and M or K 1 - 4 = Person code. M1 means interview person number one (male) and K1 means interview person number 1 (female).

The interpretation of data in the phenomenographic research begins already during the interviews. The respondents’ reactions and feelings *how* they understand and experience a phenomenon mediate knowledge, which is important to notice as well as what the respondents say. These messages during the interviews facilitate the understanding of the data as a whole. In order to achieve a general picture of the collected data I listened to the tapes, watched videotapes and read transcribed protocols several times. In the transcribed protocols, I found that some conceptions were more frequent than others and details and patterns could be identified in the interviews. I split up the protocols into four categories of description. The categories of description are considered as a main research result in phenomenographic investigations.

'Conception' is the most central concept in phenomenography (Marton & Booth 1997). 'Conception' is defined in literature in many different ways. In this investigation Sfards' definition of 'conception' is used, i.e. persons' subjective conception of an object or a phenomenon (Sfard 1991, 3). Conceptions are regarded as part of teacher knowledge (Grossman 1990).

Results

The teachers proposed different purposes for algebra/equation teaching. Their conceptions have been classified into four qualitatively different categories: (1) Pupils should learn to use equations as a tool in problem solving, (2) Pupils should learn to use equations as a tool in everyday life, (3) Pupils should learn equations in order to express their thoughts from a general point of view and (4) Pupils should learn equations in order to achieve the goals of the mathematics curriculum. The first two purposes of teaching have a practical aspect. They stress a procedure rather than the innermost ideas of equations, which is expressed in the third conception. The fourth purpose has an aspect, which is directly related to specific curriculum demands in mathematics. A more careful description of the categories now follows.

The first conception illustrates the purpose of teaching as instructing pupils how to use equations as tool in problem solving.

Conception 1: Pupils should learn to use equations as a tool in problem solving

Pupils should learn to use equations in text problems. (I3, p1, M1)

they should use equations as a tool in problem solving. (I3, p1, K2)

It is simply as a tool in mathematics. I emphasize that pupils should use equations. (I3, p1 M3)

The conceptions above indicate the practical aspects of equations in problem solving. During a lesson one of the experienced teachers gave the following practical problem. A family consists of four members: two girls Eva and Anna, mum Ulla and dad Kurt. Eva is 10 years. Anna is x years. Ulla is 5 times older than Anna plus 3 years and Kurt is 6 times older than Anna minus 1 year. How old are Anna, Ulla and Kurt if Kurt and Anna are the same age as Eva and Ulla? A teacher was very careful when reading the text problem. She asked the pupils, "What do the words 'and' and 'the same age' mean?"- Then she wrote an equation together with pupils on the blackboard (V4, K4). One of the pupils asked if it is OK to use 'ö' instead of 'x'. After solving the equation with 'ö', the teacher says, "Even if there is no 'ö' in your textbooks, you can find a lot of equations there." (V4, K4)

Some teachers also feel that they cannot realize their ideas and purposes of teaching. One of the experienced teachers complains: "I cannot teach in the way I want,

because I must be 'a police'. Today I cannot implement the things that I really want. I often lose the main thread during lessons." (I3, p1, M3)

The second conception illustrates the purpose of teaching as instructing pupils how to use equations as tool in everyday life.

Conception 2: Pupils should learn to use equations as a tool in everyday life

Pupils should use equations as a tool and they shall see the value of equations in everyday life, for example in connection to calculation of percentages. (I3, p1, M1)

Pupils should see that equations are not only 'hocus-pocus' formulas, which can be performed. They should see that equations have to do with reality. (I3, p1, M2)

Pupils get a picture of an equation like Pythagorean theory and see the use of it. They see that x may stand for a side and they get a connection to reality. (I3, p1, K4)

The pupils should get a picture, a real picture, e.g. a telephone bill in which you pay a fixed fee plus a fee for the calls you phone. (I3, p2 M3)

These conceptions stress the practical aspects of equations. They stress a 'real picture' aspect of equations in everyday life. The conceptions have a process character. One of the newly graduated teachers says during a lesson in mathematics: "A balance is equality. Equations are also balances. A balance can be an equation." (V1, K1). An experienced teacher who also uses a balance in order to give a real picture of equations says: "I use a balance in teaching of equations, sometimes I also draw." (V4, M4). He gives the following example. We do not know the weight of this stone. We are going to find out the weight by using a balance and different weights (2 hg and 0,2 hg). When the balance was achieved the teacher compared the two sides of the balance with an equation. "An equation has two sides like a balance and they are called the left-hand side and the right-hand side and in this case they are equal.....In an equation we have one letter, one unknown. The most usual is x . The weight of the stone is called x ." (V4, M4). The teacher shows that in the left-hand bowl there is a stone and one weight (2hg). In the right-hand bowl there are tree weights (2hg+2hg+0,2hg). The bowls are in balance, the equation $x + 2 = 2 + 2 + 0,2$ can be written. The teacher describes carefully how to solve the equation by using the formal solving procedure.

Examples above illustrate that the teachers try to give a real picture of equations. By using a balance, a telephone bill, applications to calculation of percentages or geometry they try to give to pupils a picture of equations in real context.

The third conception has to do with central ideas in algebra.

Conception 3: Pupils should learn equations in order to express their thoughts from a general point of view

It's important that the pupils can express their thoughts generally (I3, p1, K3)

In this conception ideas like 'using letters in algebra' and 'understanding of algebraic structures' have a central place. The following example from a mathematics lesson is illustrative. A newly graduated teacher says in a lesson in mathematics: "*It is important to learn carefully from the beginning. In this way we can promote knowledge.*" (V1, K1). She wonders, if the pupils have worked with equations already at the primary school. She writes on the blackboard:

$$1 + 1 = 2; 1 + \square = 5; 1 + \nabla = 5; 1 + \bigcirc = 5$$

She asks, "*if the pupils recognize this, adding: You have started to use equations already in the first class, but you do not know the very name.*" (V1, K1). She points out that different symbols for an unknown factor have the same meaning. She writes on the blackboard $3 + 4 = 7$. She points to the equals sign and says. "*This is called equality.*" She writes again, $2 + 2 = 4$. She asks the pupils: "*What is this?*" The pupils answer: equality. She continues to write, $x + 2 = 7$ and she asks: "*What do you call this?*" The pupils answer: equality. The teacher says: "*The equality has another name. It is called an equation, where the left hand-side is equal to the right-hand side.*" The teacher gives another example and stresses that the pupils need to learn a method for solving equations in order to solve more complicated equations. She begins with a simple equation. She writes;

$$x + 2 = 7$$

which by using other symbols for 2 and 7 can be written

$$x + * * = * * * * * *$$

In order to solve x you eliminate the two * * from both sides

$$x + \cancel{*/} = \cancel{*/} * * * * *$$

$$x = 5$$

The teacher stresses: “*You should think of a balance when you solve equations. Whatever you do on one side of the equation, you should do the same on the other side.*” (V1, K1)

After this she solves the equation above by using the formal method and continues with more complicated examples.

The example above from the lesson in mathematics illustrates that algebra is more than x and y . The example shows that a letter in algebra can stand for different symbols. The symbols like \square , ∇ and O are logically equivalent to x .

Furthermore, the example shows that algebra is more than procedures ‘to solve’, ‘to find out’ or ‘to do something’. It also includes structures. Comparing arithmetical equalities with algebraic structures can be the first step for a learner in transition from arithmetic to algebra.

The fourth conception of the purposes of teaching equations at the compulsory school has to do with the specific goals in mathematics curriculum. The goals define the minimum knowledge to be attained by all pupils after they have ‘passed’ the fifth and ninth grade at the compulsory school (The Swedish Board of Education 2000).

Conception 4: Pupils should learn equations in order to achieve the goals in mathematics curriculum

The aim is that all the pupils should achieve the goals related to mathematics curriculum. The aim is to teach so that all can at least get the mark ‘approved’. (I3, p10, M4)

The same teacher points out: “*My duty is to teach equations because all the pupils in principle must go to upper secondary school.*” (I3, p1, M4)

The teacher’s conception “*that all can at least get the mark ‘approved’*” probably considers the specific goals in the Swedish curriculum in mathematics, which make clear what all pupils should have learnt after they have ‘passed’ the fifth and the ninth grade at the compulsory school. At each school and in each class, the teacher must interpret the national syllabuses and together with the pupils plan and evaluate teaching on the basis of the pupil’s preconceptions, experiences and needs.

Discussion

The teachers interviewed have proposed different purposes or aims for the teaching of algebra/equations. According to the Swedish curriculum in mathematics the teaching of algebra should aim to ensure that pupils develop their knowledge and ability both to *understand* and to *use* basic algebraic concepts, expressions, formulae, equations and inequalities *as a tool in problem solving* (the Swedish Board of Education 2000). Many of the interviewed teachers stress that the pupils at compulsory school should learn to *use* the concept of equation as a tool in problem

solving and in everyday life rather than to *understand* the concept as an abstract entity. The first two purposes of teaching algebra have a practical aspect in the meaning of ‘a tool’, ‘to use’ and ‘a real picture’. They stress a procedure rather than the innermost idea with equations, which is a characteristic of the third conception in the study. In this the aim of algebra teaching is more than the learning of procedures ‘to solve’, ‘to find out’ or ‘to do something’. The third conception includes mathematical structures. Comparing arithmetical equalities with algebraic structures can help a learner to take the first step in transition from arithmetic to algebra. The fourth purpose of algebra teaching has an aspect, which is directly related to the specific curriculum goals in mathematics. These goals define the minimum knowledge to be attained by all pupils in the fifth and the ninth year of school (The Swedish Board of Education 2000). According to the curriculum in mathematics, the pupils should by the end of the fifth year be able to discover numerical patterns and determine unknown numbers in simple formulae in introductory algebra. Similarly, the pupils should by the end of the ninth year be able to interpret and use simple formulae and solve simple equations (The Swedish Board of Education 2000).

In my view there are several possible explanations why many teachers do not teach mathematics with focus on conceptual understanding. One is that many textbooks do not give enough support for such instruction. Another explanation is that teachers may not have sufficient competence in conceptual knowledge in mathematics. Furthermore teachers may not have had enough opportunities to develop their competence in pedagogical content knowledge. This may be due to deficiencies in school politics, educational research and teacher education. Another explanation could be the lack of understanding of the extensive knowledge that is required to teach any subject matter area. In educational research there are still few studies, which contribute to the development of teachers’ competence in pedagogical content knowledge in subject-specific area. There is also the following problem: to what extent do the research results from educational studies reach teachers, teacher educators and decision-makers? In teacher education many studies indicate that student teachers do not develop sufficient competence in pedagogical content knowledge in order to teach with focus on conceptual understanding (e. g. Tirosh et al. 1999). Perhaps because many teacher educators lack both their own research experiences and the necessary competence in pedagogical content knowledge and therefore they are not able to give student teachers the opportunity to develop such competence. To develop pedagogical content competence requires both time and resources. However, both of them seem to be an article in short supply in mathematics education.

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THE LEARNING PORTFOLIO AS AN ASSESSMENT STRATEGY IN TEACHER EDUCATION

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Abstract: *We present the design and first results of a case study about the use of the learning portfolios in the initial education of Compulsory Secondary Educational mathematics teachers. We introduce some initial sections describing the teaching educational proposal and the evaluation principles that let us to clarify the use of the learning portfolios.*

Keywords: assessment, learning portfolio, teacher education.

Presentation

The present work forms part of a research project whose principal objective is to characterize teachers' professional development through a study of initial teacher education proposals as they are put into practice in a postgraduate course. In the analysis of these learning processes, our interest is in how the educational strategies and resources that are used influence the ideas of the prospective teachers. To this end, we designed, developed, and evaluated various Educational Proposals which had the ultimate goal of bringing the prospective teachers' conceptions to the fore and mobilizing them. The methodological backbone of these proposals is the stimulation of students' reflection on their future activity as mathematics teachers and on the problems they will have to confront in that activity.

One of the professional skills that we laid particular stress on in these designs was the students' reflection on the development of their own professional knowledge. One of the strategies used within the proposed feedback/assessment system was for the students to build up a "Learning Portfolio" as their course progressed (Kelly and Lesh, 2000). The aim of this strategy was to encourage reflexive processes directed towards the identification and evolution of the students' ideas about mathematics teaching and learning (Brockban, 2002; Scherer and Steinbring, 2003).

We here present some of the first results of this use of learning portfolios in the initial education of Compulsory Secondary Education mathematics teachers while they are developing a postgraduate course. This course takes two hundred hours of pedagogical and didactical content, sixty of them involving mathematical didactical content. The unit of investigation is in its totality the mathematical didactical course. In particular, the results to be presented correspond to the reflections actually provided by the students in their learning portfolios (not required for admission to

work) with respect to clarifying a problem's meaning and to problem solving as a strategy for mathematics education (Doty, 2001).

To put the information into context, we shall first present the underlying principles of a teacher education proposal in which we use learning portfolios as a strategy for monitoring the process.

The Teacher Education Proposal

The professional profile that we are trying to aim for is coherent with the idea of the teacher-investigator. In this sense, our theoretical referent is characterized by considering the mathematics teacher's professional development to be linked right from the start to reflection on educational practice and the processes of curricular innovation (Azcárate, 2001). Reflection is therefore one of the professional skills that we lay most stress on. We regard the stimulation and encouragement of reflection during initial teacher education to an educational goal that invites investigation into its situation in practice. We therefore designed an educational proposal that permits and encourages the prospective teachers to reflect on:

- *Knowledge "of" and "about" school-level mathematics*
- *School-level mathematics teaching and learning*
- *The process of constructing their own practical professional knowledge*

The educational content is therefore not organized according to criteria of the discipline. The content and work in the teacher education classroom are organized according to the various situations, problems, and professional activities linked to the task of planning intervention in the secondary education classroom. We chose this task for initial teacher education since it requires the students to consider all the elements that are involved in their future educational practice. The methodological strategies that we designed are based on the necessary active role of the students themselves — they are responsible for their own professional learning. The strategies aim to promote in-group communication and analysis of their reflections by means of critical debate on the proposed learning situations.

To this end, work in the classroom is in small groups, always followed by processes of study and discussion to facilitate the sharing and comparison of ideas. Individual study and reflection prior to the classroom work is also important. Lastly, the instructor complements the process with moments of synthesis and closure in the form of dialectic on how the process developed.

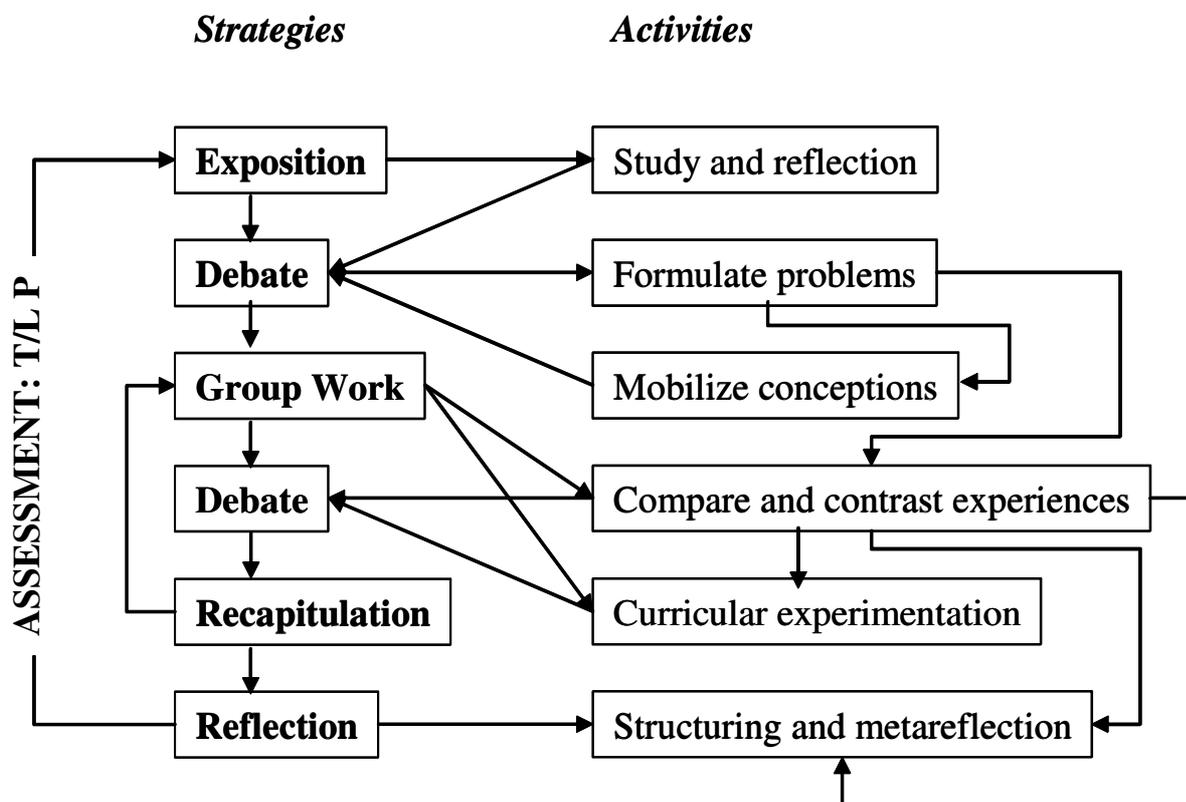
The diagram (next page) shows the organization of the educational process, and the type of activities that are proposed at each stage. This form of proceeding is repeated cyclically throughout the course, from different points of departure.

These correspond to the four groups of questions on the task of Planning the Intervention:

*What mathematical content to select?, What are the students' characteristics?
How to work up this content in the secondary education classroom?How to*

regulate and assess the instruction process?

In this type of proposal, the role of the instructor is: (i) to stimulate the formulation of suitable mathematical situations as starting points; (ii) to provide new information; (iii) to orient reflection and debate; and (iv) to open the way to new questions that will allow a new cycle of the development of professional knowledge to be initiated.



Throughout the process, the students need to become aware of their own ideas, and to understand the meaning of the new ideas that are analyzed. Study, debate, and reflection on these ideas can stimulate the evolution of their initial conceptions.

Monitoring and regulating the process

One important part of educational processes that often determines the other actions associated with intervention is assessment. There of course exist many views of assessment. To regard assessment as being just a confirmation of the goals attained by the students at the conclusion of the process would be close to reducing the instructor's mission to one of verifying the achievement of those goals and then reporting on the comparison with the objectives that were initially set out.

In our view, the purpose of assessment is to collect the essential information required in decision making (Cardenoso and Azcárate, 2003). To aim at being systematic, our assessment should be coherent with the methodological principles underlying our proposal, which is likely more consistent with a qualitative, constructivist, inductive tradition (Ryan and Kuhs, 1993; Tillema, 1998). We understand assessment to be a process that is begun on the first day of the course when instructor and students first

come into contact — they negotiate the different elements of the design, and fix their levels of demand and involvement. This type of assessment is maintained throughout the course by means of continuous review of the process. It therefore refers to the different moments and situations that arise in the classroom during the process, and should include both the assessment of the students and that of the process itself.

We therefore speak of Diagnostic and Educational Assessment, in the sense of a process which is integrated into the methodological development and that should permit assessment of the students' learning as well as monitoring and regulating the educational proposal itself. As such, it is carried out within the classroom, and is aimed at the overall improvement of the process. Since it is the most significant instrument of regulation, we shall next describe the Learning Portfolio that each student elaborates.

The Learning Portfolio

The Learning Portfolio is made up of a collection of the students' work reflecting the progress of their individual or within-group efforts in solving the activities that are set during the course, and in developing their professional knowledge. This information can provide evidence about the evolution of how they reflect on their own work (Marina et al. 2000), and the professional growth (Rolheiser and Schwartz, 2001). Given the proposal described in Sec. 2, the documents making up the Portfolio are the activities of reflection that the students carry out during the course in response to the initial problem that was presented to them.

These documents are:

- **Activities involved in the Planning tasks**, whose purpose is to analyze the different planning proposals for intervention in solving a problem.
- **Activities to stimulate comparison**, whose purpose is for the students to compare and contrast their reflections and actions with respect to the planning of the intervention.
- **Activities of structuring and metareflection**, whose purpose is to analyze the processes of reflection on planning of the intervention.

The activities involved in the Planning tasks are the basis of the educational process, and essentially constitute the Portfolio. The said process starts with each student formulating a problem situation whose solution involves school-level mathematics knowledge. The students then have to elaborate a proposal for intervention in the secondary education classroom. The planning proposal for that problem situation will gradually take shape over the course of the educational process, with the configuration of the planning of the four questions/cycles oriented to treating the problem in a secondary education mathematics classroom.

These activities are all progressively incorporated into the Portfolio as the proposal for intervention in the classroom advances. The activities are checked by the instructor and returned with the corresponding comments or indications. Beginning with the first activity, which was the formulation of the problem situation, the

Portfolio is configured by the activities that allow answers to be given to the following questions:

- *What mathematical content to teach?*

The purpose that is attributed to the process of solving the selected problem. Specification of the objectives and selection of content. Formulation of the assessment criteria.

- *What are the potential difficulties in learning?*

Description of the pupils' forms of learning, and of the difficulties and obstacles that might arise in association with solving the selected problem situation.

- *What educational strategies, activities, and resources to use?*

Description of the phases in solving the problem. Roles of teacher and pupil in secondary education in solving the problem.

Methodological strategies to follow to favour the solution of the problem. Selection and organization of the activities, materials, and resources required for the solution.

- *What, how, and when to assess?*

Characterization of what to assess, and how, why, and when to do so.

These activities are intended to respond to the four questions/cycles on planning, which are now specific for each selected problem situation and mathematical topic. A crucial part of this learning cycle involves thinking and talking or writing about the learning, reflecting (Murphy, 1998). During the teaching-learning process, the presentation and assessment of these activities provides information about the difficulties in the construction of professional knowledge, and about the different perspectives that arise in the classroom and how they evolve (Serradó and Azcárate, 2000).

This information permits the instructor to organize his or her intervention according to the prospective teachers' educational needs. Indeed the learning portfolios have enough flexibility to accommodate to the students' needs. For the prospective teacher, the content of the Learning Portfolio becomes a single activity of reflection on the planning task and the educational elements involved in it. The student's contributions to the portfolio have to be coherent with the preceding contributions. It means that the documents of the learning portfolio are both receptacles and vehicles for individual reflection (Murphy, 1998). At the end of the process, the students prepare Presentation Portfolios. In these, they select the activities of the Learning Portfolio that were most meaningful for their learning. The Presentation Portfolios have to include a list of the selected activities, with an indication of why they were chosen. That the preparation of this portfolio is a way of involving prospective teachers in the analysis of the content of the work they have carried out, and developing and understanding of what constitutes good work (Delandshere and

Arens, 2003). In preparing the portfolio, they are obliged to reflect on what use doing each of the activities of each cycle has been to them. The learning portfolio serves to document teacher growth and achievement over a specific time period (Smith and others, 2001). The reflections included in the Learning and Presentation Portfolios constitute the basic information gathering instrument in this case study design of our project of investigation.

The research design

In this sense, the Case Study design best fits the characteristics and purposes of the proposed investigation (Rodríguez, Gil and García 1999). The case study developed by the three authors, that are members of an investigation group, involved with the study of the training and the professional development of the teacher. In this sense, the authors design and evaluate the use of training strategies that could allow the professional development. In particular, one of the investigators is also an instructor. These purposes are specified in the formulation of the following problems: Which are the expectations of the instructors and students at the beginning of the educational activity? Which expectations are satisfied by the use of learning portfolios? Which are the reflection processes stimulated by the use of the portfolios? Which obstacles and problems appear with their use? Which changes in the student does their use lead to?

The study sample was taken from two courses of initial secondary teacher education that averaged 30 students per classroom. The study design included certain strategies by which the relevant information was gathered. In this sense, the proposed system of assessment which is the focus of the study is configured by the information obtained using a complementary set of information gathering instruments that provide an overall picture of what is happening in the classroom. These instruments are: systematically logged observation, the students' class diary, the instructor's diary, the individual and in-group activities, the Learning Portfolio, and a self-assessment questionnaire on the educational activity.

The process of analysis in our work is directed towards a search for meaning in order to understand the case under study. We attempt to give meaning to the information by studying and reflecting on the data through an interpretation based on assumed conceptual schemes. The analysis begins by organizing and reducing the available data. The reduction was carried out by establishing categories that reflect the centres of interest of our study, and that allow the units of information to be identified and classified.

This Category System arose from particularizing that described by Cuesta (2004) for a case study of an educational activity targeted at novice teachers, and which is based on the same educational principles.

<i>1. Development of the educational activity</i>	1.1. Expectations with respect to the activity 1.2. Processes of reflection carried out the activities 1.3. Process of elaboration and monitoring of the portfolios 1.4. Obstacles and problems in the process of their use	1.2.1. Planning activities 1.2.2. Structuring and metareflection 1.2.3. Activities to stimulate comparison
<i>2. Changes in the student with respect to</i>	2.1. Conceptions about educational practices 2.2. Educational expectations 2.3. Planning the intervention 2.4. Assessment 2.5. Problem solving	

The breadth of this Category System allows one to analyze and present general results on the use of learning portfolios in the teacher education process and their effect on the student. The present communication will deal only with the reflections collected by the students in their Presentation Portfolios (the last activity of the Learning Portfolio) relative to the intervention planning task. We selected the reflections that dealt with choosing a problem situation. The data to be presented reflect the prospective teacher's viewpoint.

Processes of reflection on the activities of curricular experimentation: Viewpoint of the prospective teacher

In their Presentation Portfolio (P-P), the prospective teachers describe their reflections on what they have learnt from each of the activities included in the Learning Portfolio, in particular, from the activities involved in the curricular planning of the solution of a problem situation. These reflections refer to various aspects: clarification of the meaning of a problem, analysis of what aspects should be planned and assessed, how teaching units associated with problems can be constructed, and how the use of the Portfolios allows ideas to be reconsidered in a process of improvement.

The clarification of the meaning of a problem or problem situation is an obstacle that the prospective teachers have to overcome, and is a priority objective in the process of teacher education. In this respect, in their Presentation Portfolios the prospective teachers say that the process followed and the use of learning portfolios allowed them to reflect on the different conceptions about the presentation of a problem:

"I have learned what a problem represents for pupils, and to differentiate it from a drill exercise or a mere application of the theory" (P-P of M^a Carmen).

As well as on the meaning of the problem, the prospective teachers reflected on the choice of problems as the initial strategy in planning their use:

"Because I have been taught, in the first place, to look for a problem and see that it fulfills the characteristics of a problem" (P-P of Diego).

Reflection on planning their use is not limited to the search for suitable problems, but also considers the form in which the problem should be put to the pupils as a function of the established purposes:

"I have learned what a problem really is, what the purpose of a problem should be, how to present a problem [to the pupils]" (P-P of Ana María).

The process of reflection on the purposes of a problem is a constant for several students, obliging them to think about a particular aspect of the planning:

"I also chose (as more meaningful) the activity of the problem because up to now I had never stopped to think about what I was going to evaluate in setting a problem. I simply expected the solution. And I believe that now, in fact I am already trying it out, before setting or proposing an activity or problem I try to plan the objectives, the difficulties they are going to encounter, etc." (P-P of Francisco Javier).

Other prospective teachers focused in their portfolios on different aspects of the process of reflection:

"To reflect on the strategies that could come in better for the pupils to solve the problem" (P-P of Ana).

"To organize the class sessions necessary for the solution of the problem, and distinguish the different types of problems and the roles of the teacher and the pupils" (P-P of Dolores).

The processes of reflection of some students, however, focused on the search for and evaluation of the different processes of planning associated with the solution of a problem situation:

"I regard this activity as very complete, since it goes back over and analyzes in a single problem the different aspects that we have been seeing in class: objectives, content, conceptual map, difficulties and obstacles, phases, methodological strategies, assessment" (P-P of Ester).

"It has been the attempt to put all of the points of an overall planning task into a specific activity" (P-P of Mario).

"Because, thanks to this [activity], without being aware of it we have been constructing step-by-step a teaching unit and all the planning that the teacher has to do to put it into practice in the classroom" (P-P of Irene).

As well as the processes of reflection on what was learnt in the module, the prospective teachers express the connection with their conceptions about the pupil's role in the proposed methodological framework:

"Going back over the problem several times to improve it allowed me to

learn to correct the roles that I had from my process when I was at school, to become aware of the difference between problem and exercise, and to apply what we saw in the classroom to a real case" (P-P of Sonia).

And since the processes of reflection constitute an instrument for the teacher's professional development:

"For me, it [planning the intervention] was the best of all because of the way it was presented to us since, as it was returned time and again as we advanced in the material of the Module, it allowed me to learn how to prepare a Teaching Unit, and also taught me that one can go on improving little by little by reflecting on my own work" (P-P of Fernando).

Conclusions

These first findings of the investigation indicate that the use of the Learning Portfolio as a system of regulation of the educational process is a powerful strategy to encourage the students' reflection, and to have an impact on their ideas. The portfolio processes provides a model for pre-service teachers to use in the learning and assessment of their own students. We also believe that the analysis of the overall case study data (from the entire set of instruments) will contribute information about evaluating the system developed for the assessment of the educational material.

The application of the Learning Portfolio as an Assessment Strategy requires a profound change in the conception that the prospective teachers have of assessment as a professional task. In most cases, there still predominates in the mind of the prospective teacher a picture of assessment as a mere statement of accounts, a confirmation of achievements, and not an analysis of how the teaching-learning process has developed. For the instructor, the Learning Portfolio permits the assessment of the development of the student's professional skills, as well as being an instrument for self-assessment. It represents a unique opportunity for the students to reflect on the meaning of their own ideas about assessment.

It is also important to mention some drawbacks. The first one is the time necessary to allow a feedback with the prospective teachers. The second one is that we should investigate if the students' reflections are really the presentation of their beliefs or the introduction of the correct answers to have a positive qualification. The third one is that to have only ten weeks of course difficult the possibility to analyse the consolidation of the students' progress.

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VALUES IN MATHEMATICS TEACHING: HOW MATHEMATICAL?

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Abstract: *The issue of values has been a longstanding concern of mathematics education research. In this paper we consider the contribution of the mathematics education literature and of the more general values education literature to our understanding of the way in which mathematics teachers embed values in their practice. A distinction which marks out mathematical values from others is a useful analytical tool, but we suggest that teachers are influenced by the interactions between those values which relate to their subject identity and their more general pedagogic or societal values orientations. Professional development approaches to values need to take account of these interactions.*

Keywords: Beliefs, values, professional development.

Introduction

A researcher interested in values in mathematics teaching might look to two main sources of work. The first is a large body of literature going back more than twenty years produced by mathematics education specialists. This body of literature begins with work on teachers' beliefs about the nature of mathematics and about teaching and learning mathematics (Ernest 1985, Lerman 1983), moving on to studies of the relationship between teachers' practices and their beliefs (Thompson 1984, Skott 2001), and most recently to research and development studies of the impact on teachers' beliefs of programmes of professional development (e.g. Cobb et al 1990). This body of knowledge has been brought together with Bishop's work on mathematical enculturation (Bishop, 1988) to inform a major project on values and mathematics teaching (Values and Mathematics Project, VAMP) which has recently been undertaken in Australia (Bishop et al, 2001, Clarkson et al, 2000, Seah and Bishop, 2002). The earlier work focused mainly on ways of examining and analysing the beliefs and values rather than looking at how these are embedded in practice, although there has been some interest in examining the extent to which espoused beliefs are discernible in practice. The VAMP project has engaged teachers in using researchers' observations of their lessons as a tool for considering the values basis of their teaching practice.

The second source is a body of work on values education in schooling in general, rather than in mathematics in particular. This work ranges from philosophical approaches to the nature of values, and in particular which values it is appropriate for

a school to attempt to transmit to its pupils, to reports of curriculum development projects which aimed to improve the schools' and teachers' effectiveness in this area. This literature includes discussion of the role of education in values formation and the values underpinnings of the curriculum. It also addresses the nature, origin and consequences of teachers' values orientations. However, as Graham Haydon (1997) has pointed out much of the literature on educational values is unacceptably vague. The literature is frequently general, aspirational and places on schools in general and teachers in particular unrealisable expectations. The literature pays relatively little attention to the ways in which teachers' classroom encounters with learners provide opportunities for values-engagements, or the ways in which value orientations of teachers may be embedded in their classroom planning, thinking and discourse.

Secondary school pupils spend the vast majority of their school hours in subject lessons, and many secondary teachers see themselves primarily as teachers of their subject. This has led us to focus on values education through subject teaching. We have chosen to do this in the context of two subjects which are generally seen to be contrasting in terms of their values stance. Mathematics is often seen as an objective, value-free mode of thought, though this is increasingly shown to be a false view (Malvern, 2001, Sinclair, 2001). However, Bishop (1988) has advanced the argument that mathematical education can be fundamentally viewed as a process of enculturation into a mathematical community, rejecting the concept that mathematics can be regarded as a values-neutral subject. Bills (2000) has explored processes of enculturation in mathematics in pre-university classes. By contrast history frequently justifies its place in the curriculum by reference to its implicit aims, including the promotion of tolerance, the ability to understand the point of view of others and to evaluate evidence objectively (Knight, 1987, Slater, 1995, Husbands, 1996).

A significant element in the values debate is a concern for the values dispositions of teachers themselves. Teachers, of course, frequently bring powerful commitment to values dispositions and orientations to their classroom work, and these may be closely related to their identities as subject teachers. There has often been a disconnection between the literatures on values education and on teachers' pedagogic practice, so that of Gudmundsdottir (1990) is significant in straddling this divide. Beginning by arguing that "although values guide teachers' practice, too many researchers believe that values are "slippery" and "unscientific....and choose to avoid [them]", she goes on to show that "the act of teaching is saturated with values both explicitly and implicitly" (1990, 45). For her, values constitute an essential component of teacher's conception of subject – an element in their pedagogic content knowledge which underpins their organisation of subject matter, rather than an element of their intentions in respect of their influence on pupils' values. Veugelers argues that the ways in which teachers deploy these values can best be identified inductively from classroom practice: "the values teachers find important for their students are expressed in the content of their instruction and in the way they guide the learning process" (Veugelers, 2000, p 40).

Within the two literatures we have been exploring there is some consensus on distinguishing between different sources of values. For example, Veugelers and Vedder (2003) develop Gudmundsdottir's insights in exploring the different influences on the values base of classroom practice. They identify three principal sources for values orientations: what they call "central" societal – frequently regulatory – values, the school's cultures and norms and teachers' own subject values. Clarkson et al (2000), writing in introduction to the VAMP project, distinguish between "general societal values often reflected in what we expect schools in general to be about, mathematical values that arise from the discipline of mathematics, and mathematics educational values that arise from the situation of teaching mathematics in school classrooms" (p1). Halstead and Taylor (2000) distinguish between those values which are intrinsic to particular subjects and those which are more general. Although these distinctions are a convenience for the theorist and the researcher, the extent to which they are real or useful for the teacher remains unexplored.

There is an old adage which claims that 'secondary teachers teach subjects; primary teachers teach children'. One of the prejudices which lies behind this statement is that it is not possible to be doing both. A teacher who is focusing on the teaching of her subject will not be also concerned with the child's development in a wider sense. This belief is compounded by a tendency for government and curriculum authorities to address issues of secondary pupils' wider development outside of subject boundaries, encouraging teachers to see pupils' wider development as something separate from their cognitive development within the subject. The mathematics education literature has, arguably, been guilty of continuing this separation by being concerned only with values which are specifically mathematical. Clarkson et al (2000) for example explain that "We have chosen to focus on those values that are associated more with mathematics and mathematics education rather than general societal values" (p1).

Our current project and data collection

Our concern within this project, then, is with the relationship between values and classroom practice in the context of subject teaching. We have chosen to focus on what Veugelers (2000) calls "the way (teachers) guide the learning process" (p40) and on the way in which values are 'embedded' in these practices. We aim to examine the relationship between subject teaching and values by starting from the teachers' classroom practice. Specifically we hoped to answer the question: 'How do subject specific and general societal values interact to inform teachers' pedagogic practice in mathematics (and in history)?'

Our analysis will distinguish between general societal values, general pedagogic values and subject specific values. We recognise the unwarranted simplicity of the notion of 'general societal values', but find it a useful label for those values which concern teachers because they are interested in their pupils' development as members of society, rather than as learners, or more specifically as learners of their subject.

General pedagogic values are concerned with approaches to learning. By subject specific values we mean those values and principles which underpin a school subject as a body of knowledge and set of agreed procedures (White, 1982).

The project has collected data from both mathematics and history teachers and used these data to contrast the two subjects, but in this paper we consider only the data from the mathematics teachers.

We chose eight research participants, four mathematics and four history teachers. They all taught at non-denominational, mixed comprehensive schools, which represented a range in terms of pupils' socioeconomic backgrounds. The teachers were all considered successful by their head teachers and departments, and had a range of length of teaching experience, from two years to more than thirty years in the classroom. Four were heads of department or faculty and four were female.

Our concern with classroom practice led us to begin our data collection with observation of teaching. In each case one of the researchers observed the teacher over a full day's normal timetable, seeing at least three hours' teaching and three different classes. The teachers had only the barest advance information about the purpose of the observation and were asked not to make any adjustments to their plans in view of the observation. The teachers wore a microphone throughout the lessons for the day and the resulting recordings were transcribed.

The second stage of the data collection was to interview the teachers in order to hear them talking about their understanding of the situations we observed and their perceptions of the actions they took and the decisions they made. The questions that we asked in the interviews directed the teachers' attention to particular incidents in the lessons and asked for their views. These incidents were sometimes related to the teachers by playing back the lesson tape, sometimes by reading out a part of the transcript, and sometimes by the researcher's description.

In order to select the incidents to present in this way we made a preliminary analysis in which we looked for teaching actions which, on the basis of one day's observation, we might suppose form a regular part of the practice of our research participants. We refer to them as gambits to reflect their deliberate but often small-scale nature, and to distinguish between the action and its rationale. There is inevitably a degree of subjectivity in the identification of these gambits, and we do not claim that these are in any way representative of their practice as a whole. However, we have tried to identify teaching actions which were repeated (in the sense that a response which was similar in the researcher's eyes was made in a similar situation) at least once during the day, and those which are reported here were recognised by the teacher concerned as something they did habitually. We also wanted to choose actions which were, if not unique to the teacher in question, then at least relatively uncommon. We did not, for example, identify insisting on hands raised to respond to a teacher question as a gambit. Finally we were interested in gambits that the teachers were at least partly conscious of as gambits, so that they could offer some discussion of their practice in

this respect.

Having identified gambits in this way from the tapes and transcripts of lessons, we interviewed the teachers, within a week of making the lesson observations, basing our questions on these gambits. Typically we would present a few snippets of data which, in our view exemplified a particular gambit, and ask the teacher to comment on how they had acted.

The data presented below consists of descriptions of teaching gambits identified by the researchers for each of three of the teachers and the teachers' responses to being asked about these gambits in the interview. We use these responses to raise issues for discussion and for further research, and to draw out tentative implications for practice.

Sally: Sally is a maths teacher with four years' experience at a rural 11-16 comprehensive school. She is second in charge of the maths department. During the lessons we observed Sally several times asked pupils to give answers to questions, accompanying the request with a comment that it didn't matter if the answer was wrong. We give two examples to illustrate this. In a year eleven lesson Ruth approached Sally to say that she had finished the exercise that had been set for the class:

S: You've done it, you think? Do you want to put the answers on the board?

R: Are they all right?

S: Well, we'll discuss that and decide if they are all right between us

R: I don't want to be wrong

S: (referring to Ruth's notes) You can take them with you - it doesn't matter if you're wrong, cos if you're wrong then people – you can learn from the mistake.

Later in the lesson, Sally added:

S Well done to Ruth cos she's actually got up here and volunteered, ... she was scared in case she was wrong, but it doesn't matter if she's wrong.

In a year ten lesson Sally was addressing the whole class and had asked a question which had been answered by Carolyn. Sally was beginning to move on taking Carolyn's answer as a starting point, but Tom was obviously keen to offer a further response to the original question:

S Tom, what were you going to say?

T Would you just do 99 times 3 and then add it on to 13?

S We'll see ... We'll keep that there and we'll test that out and decide if it's right.

A few minutes later, Sally returned to what she had written on the board and said:

S We're going to test out to see if Tom was right. And if he was or he wasn't, it doesn't make any difference

These two incidents illustrate Sally's habit of asking pupils to contribute answers (to exercises or to her verbal questions) which are offered for public scrutiny. During the

interview Sally was asked about the thinking that lay behind her actions in these cases. She gave an answer which ranged over concern for the pupils' emotional responses and confidence as well as interest in the pupils' thinking behind the answers they gave. The following two excerpts illustrate her response:

'I just wanted them to realize that it's okay to make mistakes and that if we flag up common mistakes that people make, then hopefully they'll learn from that and not make them again, and I was trying desperately not to make her feel bad that her answer was wrong...

(About Tom) 'I didn't want to say that's not right straight away, because he would never know why it wasn't right but also if his thinking was a way that other people were thinking ... they would see why they weren't right as well'

Sally's response is perhaps best summarized in her opening statement above, 'I just wanted them to realize that it's okay to make mistakes'. This is a values statement, where she expresses a wish to encourage her pupils to share that value. The reasons that Sally gives for this create some difficulty for an analysis of her values in terms of general societal values, general pedagogic values and subject specific values. Her wish to protect the 'face' of the pupil who is offering their work for scrutiny is inseparable from her promotion of tolerance amongst the rest of the class. Her interest in pupils' cognitive development through analysis of errors is entangled with her encouragement of a sense of community, of working to help each other to learn.

Simon: Simon is a maths teacher with about twenty years' teaching experience and he is head of faculty at a city centre, multi-ethnic 11-18 mixed comprehensive school. In the lessons we observed he frequently responded to pupils' answers by asking the rest of the class to agree or disagree. For example, he asked a year 7 class to do a mental addition and after a few moments said:

"Can anybody add up those marks? Um, Zainab. 16. Who agrees? Who agrees with 16? Okay, Nazneen what do you think it is? 15. Okay, who agrees with 15? Okay, who's got a different answer? 17 you think? Okay. I think maybe some of you need to check. Put your hands up if you think it's 17. Put your hands up if you think it's 16. Most of you. Put your hand up if you think it's 15. Okay Keith you don't, you're not thinking? You've got 16 now. It is 16, thank you"

In a year 12 statistics lesson, Simon poses a question "So does it matter that that is 'greater than' rather than 'greater or equal to'?" One pupil replies 'Yes'.

P2 No it doesn't matter because it is continuous

S Ok we have a discussion going on here. Does it matter if that is greater or equal to? Put your hand if you think yes. (3 pupils raise a hand). Put your hands up if you think no. Ok put your hands up if you think you have any evidence or explanation for what you think at all rather than just a gut feeling. Ernest – right, tell us.

Asked in interview about the gambit of looking for several answers to the same

questions, Simon said:

“The idea of not giving a right or wrong is I think partly to give value to all answers but also to make the person who is giving the answer think of other responses that might have been given. So that, they might actually be sure of their answer and if their answer is right and they are confident with it ... it's not going to sway them anyway. But they can actually see why other people have made mistakes perhaps then it might help them to avoid them in the future.

“And also to sort of engender some sort of culture of not being afraid of getting answers wrong. So the culture in the classroom is supportive of people having a go and making mistakes.”

Although the teaching action we identified was quite different from Sally's, Simon's description of his reasons for using it is similar to Sally's in many respects. He shares her concern that pupils see mistakes as useful and that the classroom culture is supportive of 'having a go'. As with Sally, it is possible to trace a number of potential 'sources' for the values that Simon is espousing. Neither teacher makes specific reference to mathematics as a source or context for these values.

Stuart: Stuart is a maths teacher of more than twenty years' standing who is assistant head in an 11-18 mainly white, mixed comprehensive school with an urban fringe catchment area. The issue that we want to pick up from Stuart's lessons was not so much a gambit as a theme. On several occasions he spoke to pupils about the way in which they presented their work. For example, in a year nine lesson, he said to a pair of pupils:

“Don't use Tippex - just cross out in maths. Elegance is about good quality crossing out”

Again towards the end of a lesson he said to the class:

“Very pleased ladies and gentlemen both with the quality of work that you are producing from the point of view of the algebra, but also may I say your setting out is superb. ... It's all very neat, very precise. Well done everyone”

Interviewed after the lessons, Stuart had a lot to say about this issue, much of which we do not have room here to report. He spoke first of his interest (which all the pupils knew about) in medieval copyists

“they cross things out so elegantly so the pages aren't ruined and that concept of being able to make a mistake and correct it I think is very, very important. And it is something that the modern world is losing because of ICT and because of people's perception of what 'good' is about.”

Referring to the comment on pupils' setting out he said:

“but we've now got to that stage of being able to say yea this page looks nice. Not because the handwriting is good ... but because the equal signs all follow one down from the other. You can give that to anybody who knew mathematics and ...

if you had made a mistake they would be able to spot it very, very quickly and easily and that's valuable of itself, a valuable piece of learning"

Stuart is more explicit than either of the other two teachers in talking about how he hopes to, and thinks that he has, influenced pupils' values. He wants pupils to value certain aspects of how their work appears, and this is particularly to do with the way in which they deal with mistakes. Stuart does set these remarks in the context of mathematics, but it is clear that his concern goes beyond this context to, for example, "people's perception of what 'good' is about".

Discussion

The three teachers whom we have quoted here each express strong commitments to particular classroom practices through which they express their espoused values. The selection of values that were discussed at interview depended both on the events of the day's observation and on the observer's interpretation of those events and we make no claim that the interviews allow us to see what are the core values driving a particular teacher's practice (or even that it is appropriate to think of values as driving practice). We have chosen the three data extracts above because of their common theme, that of making mistakes, and because they represent repeated and conscious actions on the part of each teacher and are, in that sense, typical of their practice. The common theme allows us to use these three extracts to raise issues about our analysis of the embedding of values in classroom practice and its implications for professional development.

The first issue concerns the link between subject and values. Encouraged by the literature, we have looked for clear distinctions between subject specific values and those from other sources. However, as is exemplified in the data above, it is very difficult to separate out subject specific from other values motivations in the teachers' descriptions. This is particularly so if we take the teacher's actions as the unit of analysis, rather than the justification. In other words, it is particularly hard to find practices which are justified purely in terms of subject specific values.

We might want to argue, then, that the subject being taught is largely irrelevant to the values agenda. The valuing of mistakes and the use that might be made of them, for example, is surely something that all teachers would find relevant. There are at least two reasons for not arguing in this way. The first is that, although we have not found evidence that this is widespread, there may be a lot for teachers and researchers to gain by considering what the subject specific values are that underpin their teaching of mathematics and how these are and might be embedded in their classroom practice. The second reason is that, although the valuing of mistakes is something not uniquely important in mathematics, we suggest that outworking of this value in teaching actions, and the importance attached to it depend very much on the subject context. In other words, although teachers of other subjects may place a value on making use of mistakes, the way in which they embed this in classroom practice may be significantly different.

As mathematics educators we tend to focus on those aspects of teachers' thinking about their practice which are specifically mathematical, whereas our data suggests that teachers' subject specific and more general concerns for their pupils are intermingled in their rationale for their teaching practices. Our attempts to enable teachers to reflect on the values that they embed in their teaching practice may be more effective if we take our agenda from the teachers' concerns and understandings rather than from our own expertise. In other words, we might do well, in our work with teachers, to treat beliefs and values about the nature of mathematics, its teaching and learning as just one aspect of the belief schema on which teachers draw. Skott (2004) makes a similar analysis to ours of teachers' responses to classroom data. He starts from an analytic viewpoint which is concerned with the way in which teachers are seen to bear a very heavy responsibility for curriculum enactment in line with a reform agenda (a situation which he calls 'forced autonomy'), but finds that the explanations which teachers give for their classroom actions are as much concerned with broader educational concerns as they are with mathematical learning. This leads him to conclusions which have something in common with our own, "For the mathematics education research community this implies that a broader perspective is needed than one of focusing on mathematics and/or meta-mathematics when understanding the role of the teacher in the enacted mathematics curriculum"(p. 253).

We see two immediate implications of this for research and for practice. Currently the professional development agenda in schools tends to focus on subject teaching or on whole school issues, where subject is seen as irrelevant. If teachers are acting in a way which is driven by the subject they teach, but basing their justifications for their actions on more general values, then we see an argument for engaging teachers in dialogue with both subject specialists and with teachers of other subjects. This dialogue would draw attention to contrasting values interpretations in a way which might both reinforce subject identity and values and present alternative ways of acting in the classroom.

In terms of research, we suggest that our understanding of values in mathematics teaching is constrained by distinctions made by past researchers between mathematical and other kinds of values, and in particular by decisions to focus on one kind to the exclusion of others. Our data suggests that the values basis for teachers' actions is far more complex than this analysis allows, and that any attempt to engage with teachers concerning their values orientation needs to recognize this complexity.

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FROM PRACTICES TO THEORIES TO PRACTICES ... IN LEARNING TO TEACH MATHEMATICS AND LEARNING MATHEMATICS

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Abstract: *I introduce the three ideas of ‘purposes’, ‘basic-level categories’ and ‘meta-commenting’, illustrating their explicit use for learning to teach mathematics within teacher education and for teaching mathematics. After showing how the idea of purposes emerged from my practices observing the work of student teachers of mathematics, an enactivist theoretical frame and methodology provides a link to the theoretical construct of basic-level categories. In working within this theoretical framework, the practice of ‘metacommenting’ arose. The three ideas are used explicitly at the start of courses when the teacher or teacher educator’s concern is to develop a learning culture in which their learners’ behaviours as mathematics students or student teachers can become established.*

Keywords: initial teacher education; embodied cognition; basic-level categories-

In this paper, I first link together the two ideas of ‘purposes’ (Brown and Coles, 2000) and ‘basic-level categories’ (Rosch, in Lakoff, 1987) illustrating their explicit use in the learning and teaching of mathematics and mathematics teacher education. I am interested in the development of cultures different to previous experiences: How does a person, a student teacher, leave behind the images of the teachers who taught them mathematics to begin the process of becoming the mathematics teacher they want to be? How does a student of age 11 years leave behind negative images of mathematics on transition to a new school? [2] In line with a paradigm shift from a focus on knowledge to the process of coming to know, I am interested in the mechanisms to support this process rather than any substantive findings in terms of particular subject content or teaching strategies. [3]

Purposes

I started teaching at the University of Bristol, Graduate School of Education in 1990. I had been teaching mathematics in a secondary school for fourteen years, leaving as a head of department. In the spirit of Schön (1991), I made a conscious decision to try to be aware of my practice as a teacher-educator in a way in which I had not been as a teacher. The questions that engaged me were to do with ways of working with student teachers, and those new to the profession, to develop the effective practice of mathematics teaching. My first explorations were in some senses naïve, but out of those practices grew a theory-in-action (Schön, 1991) in which I started to use the word ‘purposes’:

Beginning teachers need to temper idealistic goals given the reality of how much skill might be required to achieve them. [E]ngaging with a student [teacher] on a philosophical level [...] did not seem to allow practical development or change of implicit theories [...] (Claxton, 1996); nor did giving ‘tips for teachers’ at a behavioural level do much for their developing sense of who they might be becoming as a teacher. (Brown (with Coles), 1997, p. 104)

The dictionary (Chamber’s Twentieth Century, 1976) definition of purpose reads: *idea or aim kept before the mind as the end of effort; power of seeking the end desired*. There is nothing here about actually getting to the end, although that is not precluded. With purpose, however, we give ourselves the motivation to make effort in relation to some ‘idea kept before the mind’. Purposes emerged for me as a description of the sorts of guiding principles that student teachers found energising when learning from their own experience, seeming to be in the middle position of a hierarchy between philosophical attitudes and teaching behaviours in the classroom.

Currently, in working with student teachers at the start of their course, I state the function for the year as ‘gaining a sense of the teacher they want to become’ (Brown, 1997). Discussing incidents from their classrooms is important. To focus the discussion, one person is invited to describe in detail an anecdote from their practice and this is followed by accounts from other student teachers that seem to them to be similar in some way or feel different. After a few such stories have been shared, an invitation to label what we are actually talking about is given. These labels (*e.g.*, ways of sharing students’ responses) become the student teachers’ purposes, which allow them to begin to collect a range of behaviours to try out in their classrooms.

Theoretical frame and methodology

The methodology of enactivism has its roots in a theoretical perspective, emerging in a number of different disciplines, that views our minds as ‘embodied’ and learning as ‘embodied action’ (Varela *et al.*, 1995, Maturana and Varela, 1992, Maturana, 1987, Clark, 1997, Lakoff and Johnson, 1980, 1999, Lakoff, 1987, Johnson, 1987). Hence, enactivism provides not only a methodological stance, but also the theoretical frame that will be used in the rest of this paper. The key ideas in this framework are: embodiment, metaphor and basic-level categories.

Embodiment: Essential to this theoretical framework is the belief that we are what we do - all our past experiences contribute to our current actions - and that the way we develop the embodied actions that make us who we are is through categorising:

Every living being categorizes. Even the amoeba categorizes the things it encounters into food or nonfood, what it moves toward or moves away from [...] We have evolved to categorize; if we hadn’t, we would not have survived. Categorization is, for the most part, not a product of conscious reasoning. We categorize as we do because we have the brains and bodies we have and because we interact in the world the way we do. (Lakoff and Johnson, 1999, pp. 17-18)

In my work there is little separation between theory and practice. I see my 'research about learning as a form of learning' (Reid, 1996, p. 208) where that learning is gaining a more and more complex set of awarenesses that inform my practice of teaching both of mathematics in the classroom and in working with student teachers. In this process the act of making distinctions is the basic tool, and, as I look, I inevitably come to see more about that which takes my attention.

In my practice and research I do develop theories and models but these

are not models of. [...] They do not purport to be representations of an existing reality. Rather they are theories for; they have a purpose, clarifying our understanding of the learning of mathematics for example, and it is their usefulness in terms of that purpose which determines their value. (Reid, 1996, p. 208)

The aim [...] is not to come to some sort of 'average' interpretation that somehow captures the common essence of disparate situations, but rather to see the sense in a range of occurrences, and the sphere of possibilities involved. (Reid, 1996, p. 207)

Thought is mostly unconscious, according to Lakoff and Johnson (1999):

conscious thought is the tip of an enormous iceberg. It is the rule of thumb among cognitive scientists that unconscious thought is 95 per cent of all thought - and that may be a serious underestimate. Moreover, the 95 per cent below the surface of conscious awareness shapes and structures conscious thought. (p. 13)

This 95 percent is called the 'cognitive unconscious'. Most of who we are and most of what guides our actions is therefore in this unconscious; these embodied, automatic actions are sometimes labelled 'enactions' ('knowing as doing', Bruner, 1996). These actions embody complex decision-making processes that are largely implicit and therefore fast:

Minds make motions, and they must make them fast - before the predator catches you, before your prey gets away from you. Minds are not disembodied logical reasoning devices. (Clark, 1997, p. 1)

In the case of student teachers beginning the process of entering a new world of the classroom, without a range of effective behaviours, what seems important is that a structure for learning is put in place by the teacher educator that supports the student teachers in their move to implicit effective behaviours (embodied actions). As a new teacher you want those actions (effective behaviours) and you want them fast!

Metaphors: Our minds are embodied not in the trivial sense that we have a physical brain, but in the necessary and fundamental sense that all our thinking is grounded in our sensorimotor system and arises from our bodily experiences. Lakoff and Johnson (1999) claim that, from an early age, correspondences are established between our sensorimotor experiences and our subjective ones. For example, in our first years warmth and proximity are indistinguishable from affection. When we learn to discriminate the sensorimotor from the subjective those links that have been forged in

our neural system remain. Hence we still speak of ‘a close friend’, ‘a cold introduction’, ‘a warm greeting’ and so on. Out of these types of ‘primary’ metaphor we develop, by combination, the complex conceptual metaphors Lakoff and Núñez (1997; Dehaene, 1998; Lakoff and Johnson, 1999) claim are what we use to think: ‘[even a]bstract concepts are largely metaphorical’ (Lakoff and Johnson, p. 3) and

Mathematics thus, is not the purely “abstract” discipline that it has been made out to be [...]. Our mathematical conceptual system, like the rest of our conceptual systems, is grounded instead in our sensorimotor functioning in the world, in our very bodily experiences. (Lakoff and Núñez, 1997, p. 30)

Furthermore, as Lakoff and Johnson (1999, p. 59) argue:

We do not have a choice as to whether to acquire and use primary metaphor. Just by functioning normally in the world, we automatically and unconsciously acquire and use a vast number of such metaphors. These metaphors are realized in our brains *physically* and are mostly beyond our control. They are a consequence of the nature of our brains, our bodies, and the world we inhabit.

Through metaphors we build concepts that enable us to function and interact in the world. A majority of these interactions, they argue, are embodied and unconscious.

Basic-level categories: We develop hierarchies of categories, e.g., ‘Chino’ is a ‘dog’, which is a ‘mammal’, which is an ‘animal’. The categories that are most important in our capacity to interact effectively with the world, and around which most of our knowledge is organised, are in a middle position and are labelled ‘basic-level categories’ (Lakoff, 1987, p. 49, in the example above, the basic-level category is ‘dog’, there is too much detail in the particular dog and ‘animal’ is too abstract).

In Lakoff and Johnson (1999), basic-level categories are characterised by four conditions:

Condition 1: It is the highest level at which a single mental image can represent the entire category. (p. 27)

In the example of ‘dog’ it is possible to have a sense of one generalised mental image for that category compared to, say, ‘animal’ which would be a more abstract category because there is not one general image of an animal. Lakoff and Johnson (1999) suggest ‘car’ and ‘chair’ as basic-level categories in contrast to, say, ‘vehicle’ and ‘furniture’.

Condition 2: It is the highest level at which category members have similarly perceived overall shapes. (p. 27)

Given the single mental image of ‘dog’, how is a dog recognised in the outside world? We can recognise a dog in the outside world because there is a similarly perceived overall shape that fits with the generalised mental image. This would also, for example, apply to ‘car’ and ‘chair’.

Condition 3: It is the highest level at which a person uses similar motor actions for interaction with category members. (p. 28)

In our cognitive unconscious are automated behaviours that embody similar motor actions for interaction with any basic-level category member, such as 'sitting' for 'chair', 'stroking', 'feeding' for 'dog' or 'parking' for 'car'. There are usually no such patterns of everyday behaviour at the more abstract level, *e.g.*, consider 'furniture' where there is no single motor action which could be appropriate for interaction with all category members for most people but for a manager of a self-storage firm there might be similar motor actions for 'furniture' and hence it is possible for 'furniture' to be a basic-level category.

Condition 4: It is the level at which most of our knowledge is organised. (p. 28)

If I only learn how to mend one car I will have a lot of detailed knowledge that will not necessarily help me to mend a car of a different make. Once I engage in learning about a second make of car, I begin to learn about the properties of the general category 'car' *e.g.*, all cars have engines but not always under the bonnet! Since it is around this category that my new knowledge gets organised 'car' is basic-level.

Purposes as basic-level categories

The questions, given in the first paragraph of this paper, are to do with learners entering new worlds. How does all this theory illuminate what I do as a teacher-educator? What is crucial to being able to function effectively in these new environments is that the student teachers are able to build their own metaphors, creating their own basic-level categories linked to embodied actions through which they can begin the process of learning.

The rest of this paper illustrates how I currently work with these ideas through 'purposes' that become part of the learners' ways of staying with the complexity in the classroom and support their natural learning through making distinctions in their worlds. I will briefly show how the theory developing out of my original practices fed back in to practice again by illustrating the use of metacomments as a tool to support teachers and teacher educators in supporting the development of purposes or basic-level categories in their students or student teachers.

Basic-level categories connected with purposes for me because both hold the middle position of a hierarchy between abstraction and detail. Rosch (in Lakoff, 1987), reported that basic-level categories are 'the generally most useful distinctions to make in the world' (p. 49). Having been shown the picture of a dog we are able to recognise it as such and most people would use the word 'dog' to describe the picture. This is the 'basic-level category'. There would be some individuals who might know that particular dog and so use its name as a descriptor and be able to give details of that dog's individual characteristics and behaviours. There are also people who would describe dog in a more general, technical or abstract way. This gave some additional support for the evidence I had of teachers talking in terms of 'purposes' in what

appeared to be the most useful and most easily expressed distinctions that described and motivated their teaching actions.

My attention linked basic-level categories to purposes as ways for student teachers to cluster groups of behaviours or teaching strategies and, where a student teacher found working with particular purposes effective, this was a way of influencing their developing images of mathematics and teaching mathematics. Without my own developing practice and thinking as a teacher-educator, I am not sure that I would have been able to use the ideas of embodiment, metaphor and basic-level categories so directly in my work, but the conditions developed by Lakoff and Johnson (1999) have supported my developing theories-in-action, a developing set of theories that are 'good enough for' my practice as a teacher-educator, the practice of teaching of my student teachers, the learning of mathematics of students and my work as a researcher. My work on purposes (Brown and Coles, 1996, 1997, 2000) supported the learning of these students, teachers and student-teachers. Purposes, labels for common descriptions of experiences but not common practices, provide a structure within which student teachers can develop their practice, evidenced in lesson and course evaluations. The link with basic-level categories helped me, their teacher, to recognise 'purposes' through the sense of the simplicity of their articulation by the student teachers. I could recognise and work with their statements more effectively. It does not matter that we are not sharing common experiences of classrooms because the student teachers are developing guiding principles for action.

These theories-in-action are an integral part of who I am now and what I do. The following writing, done immediately after an interview with a potential student-teacher before the course, illustrates this:

She described the teacher starting to teach angle by asking the pupils what they knew about angle [this was a description of behaviour which I recognised as the 'detailed layer'] Her next comment 'to connect with what they already know' I recognised immediately as potentially a basic-level category. I [...] asked her to imagine other ways of 'knowing what the pupils know' [I would call this a purpose, but not explicitly to the interviewee at that stage]. We brainstormed other ways in which this could be done. [...] In recognising more than one strategy she has begun to organise knowledge around a basic-level category. As a teacher in a classroom with this purpose, there is the potential for her developing the use of a range of suitable teaching behaviours.

How does a teacher recognise an opportunity to, *e.g.*, 'share responses'? I think of the 'shape' of such a purpose to be a recognition of similarity in what the students are doing (*e.g.*, an awareness of a range of ideas being generated by their students through working on a task suggests the use of a behaviour to structure the lesson to 'share responses'). In this way an expert teacher seems to make complex decisions very quickly and does not work with a script (Brown and Coles, 2000).

Metacommenting

In my practice I was aware of being able to recognise in the student teachers their identification of basic-level categories, their motivations for action at the middle level. I was then able, as my attention was caught, to express those ideas to them, directing their attention by using ‘metacommunication’ (Bateson, 2000). In establishing a classroom culture at the start of their first year in a new school, Alf Coles, a classroom teacher and research collaborator, uses the idea of what we came to call ‘metacommenting’ to focus the attention of his students on behaviours that will support their ‘becoming mathematicians’ or ‘thinking mathematically’. A working learning culture is developed within the group that is somewhat different each year given that what students do and notice is different.

When students are engaged in an activity, Alf comments on behaviour that he notices as being mathematical. Students write at the end of an activity (usually at least four lessons) about what they have learnt both at the skills level and about thinking mathematically. Typically the students work with the language of ‘conjecture’, ‘proof’, ‘theorem’, ‘testing conjectures’ and ‘counterexample’ because this is the language used by their teacher. Metacomments are in ‘italics’; Alf speaking is indicated by ‘-’ and students by ‘~’ at the start of lines:

- There were some counterexamples to that. Remind me what that is.
- ~ One that does not fit the conjecture.
- OK, Ben has done something very mathematical. He’s *gone back and looked again and changed it* [the conjecture].
- ~ [Later in the same lesson.] All two digit numbers will add up to 99. [David’s conjecture is written on the board.]
- ~ I’ve got another counterexample to Ben’s.
- This is how mathematicians work; *are there counterexamples? Are two conjectures actually linked* and so on.

Other metacomments that Alf has used are ‘getting organised’, ‘asking and answering your own questions’ and ‘listening and responding’ (to what other students say). So, because Alf is talking about what students are doing that fits with ‘thinking mathematically’ some of these metacomments capture the attention of the students and become their purposes, giving them a sense of knowing what to do. Over time they develop strategies to work on problems and learn skills.

In both cases the students or student teachers learn what they need to and the teacher or the teacher educator is learning about learners’ strategies and behaviours as the culture develops in the group. Such learning cultures can also develop within a group of teachers working together in a school where heads of department meta-comment on ways in which they want the teachers in their departments to work.

Back to practice as a teacher educator

So, I am clear that the student teachers will be able to use their powers of discrimination to learn about teaching mathematics and that it is important to allow *them* to find teaching strategies that work to support the learning of their students. The fact that that is how I expect us to work needs setting up. I explicitly state that I do not have an image of the best way to teach. At the start of the course, the first time the student teachers have a session with their mathematics education tutors at the University, they have already spent two weeks observing and working in a primary classroom. The students are invited to bring to mind one image that seems to them to be about 'teaching' arising from their recent experiences and one that seems to be about 'learning'. Based on these real experiences they must be prepared to share with others in the group completed slogans for 'teaching is ...' and 'learning is ...'. This activity serves to orient the students to see the 'shape' of their classroom interactions and allows issues to arise that might become purposes for them at a later time. The activity serves to stop them from living at the level of reporting long held beliefs or attitudes divorced from their recent experiences. For me, however, 'gaining a sense of the teacher they want to become' is a purpose, since I have a range of behaviours as a teacher educator related to offering this task and I recognise their purposes as they are articulated or enacted and can metacomment on the range of those.

Here are a few of the statements of the student teachers of 2004/5 from that first session of the year: being aware of what my students know; making mathematics real and visual; catering for different learning styles. These statements are the student teachers' purposes. They do not know yet what to do as they teach, but will be beginning to collect a range of strategies or behaviours - that they notice through observations of practising teachers and viewing videotapes, and perhaps read about or hear others describe - to try out in their own classrooms.

Evidence from the student teachers' writings, lesson and course evaluations shows that the language they use to describe and develop their own teaching is one of purposes and actions, giving a powerful sense of their own learning and progress. The following example is taken from the assignment writing of a student teacher in the first term of the course, 1999. She has had five weeks of experience teaching two classes in a secondary school:

I wrote the question on the board, and asked for hands up for the answer. The first child gave me the answer to which I said 'Correct'. Belatedly, I asked if anyone else had anything different, but of course the children were then unwilling to offer an alternative answer that they now knew was definitely wrong. I realised immediately that I could not now see what the rest of the children had done. Since that occasion I have been attempting to gain answers from several members of the class [...] An advantage in listing the variety of answers to a question is to show children that they are not alone in making a mistake and that others have had the same (or different) problems. Similarly, multiple equivalent answers can be highlighted whereas otherwise a child may

feel that their answer is wrong just because it does not look identical or is in a different form. Hence the art, as a teacher, of 'being expressionless' as a variety of answers are given to a problem appears to be a very useful one.

Firstly, there is the experience. How did this student teacher recognise the possibilities inherent in not accepting that first correct answer. In talking to her after reading this work, she reported to me that, in fact, she had seen another teacher ask for multiple answers but that the first one offered had been wrong and the whole class had gained a lot from exploring the errors of the students. Her students' reactions to the correct answer being confirmed were different (discrimination, categorisation) and it was harder to motivate them to continue looking at other answers. This led to a change in her behaviours as she experimented with different ways of collecting responses and ends with her developing a label for these behaviours, 'being expressionless', which then becomes a purpose, linked to actions, for her. She works with this purpose, at a different level to the discourse with her students, and could begin to metacomment on the process.

Conclusion

From this enactivist position there is a direct link between theory and practice. I work with learners' actions at all levels. I give the responsibility to learners as they work on mathematics or as they work on becoming teachers. I do not myself simplify students' learning of content or direct student teachers to a particular way of teaching. I am convinced that they can learn in a complex way from the start of their courses because that is the way that we learn naturally about our world. I spend my time observing their learning and commenting on it. I can make fine distinctions about the learning of mathematics because this is what I am interesting in and have spent my time looking at. Learners develop their own basic-level categories.

Acknowledgements

[1] Many of the parallel strategies for teaching mathematics and the work of the teacher-educators and researchers have been developed with Alf Coles and/or David A. Reid. An earlier version of this paper was an unpublished contribution to an invitation conference looking at the fit between social constructivist, social practice and socio-cultural perspectives in mathematics education research, Norway, 2000.

[2] Student teacher refers to a learner on a mathematics teacher education course and student refers to a learner in a mathematics classroom.

[3] This is a theoretical paper illustrating the use of the ideas of purposes, basic-level categories and metacommenting for the practice of both the learning of mathematics and learning to be a teacher of mathematics. The research underpinning these ideas was, initially, action research on my practice as a teacher educator, followed by a longitudinal study (currently nine years) of the development of the mathematics teacher Alf Coles (observations by me of his teaching practice at most once a week).

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TEACHER'S ACTIVITY AND KNOWLEDGE: THE TEACHING OF SETTING UP EQUATIONS

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Abstract: *In this paper, we briefly present the elements of a cross theoretical framework addressing the teacher's activity, inspired by the theory of didactic situations created by G. Brousseau and the anthropological approach of Y. Chevallard. These theoretical tools are applied to the case-study of a french teacher planning and conducting an ordinary lesson of algebra.*

Key words: theory of didactic situations, anthropological approach, teacher's activity and knowledge; ordinary teaching of algebra; case study.

INTRODUCTION

The analysis of teacher activity from a didactic point of view (taking into account the specificity of mathematical content knowledge) is a current major topic, however difficult to tackle. Within international math education literature, the number of papers related to mathematics teacher practices has grown fast for the ten last years. The intricacy of «the teacher problem», as some researchers call it (Bosch 2002), lies in the multidimensional and dynamic features of teacher activity. Robert (2001) claims that teacher practices are *complex* and cannot be split into isolated units, as lessons planning, progress, etc., and *require the combination of interrelated mathematical, didactical and pedagogical knowledge*. She adds that *rearrangements take place all the time*: for example, lesson planning influences classroom management, but unexpected events might lead a teacher to change a lesson plan.

In France, researchers have developed three main theoretical approaches to examine teacher practices:

- the approach inspired by the theory of didactic situations, related to the model of *milieu* structuring (Brousseau 1997, Margolinas 1995).
- the anthropological approach developed by Chevallard (1999, 2000) and his team, related to the model of mathematical and didactical praxeologies.
- the twofold approach of the mathematics teacher's activity: didactic and cognitive ergonomic (Robert and Rogalski 2002). This point of view allows to tackle the different determinants (individual, social and institutional) of the teacher's activity and the activity of students prompted by the teacher in the class.

As other various theoretical frameworks (Schulman 1986, Even and Tirosh 1995, Artzt and Armour 1999, Jarworski 2002), these approaches aim to sort out the complexity of the teacher's activity in different ways.

In our research, we use the two perspectives derived from the theory of didactic situations and from the anthropological approach, to gain better understandings of teacher activity in relation to algebra teaching-learning. We will start by briefly outlining the models issued by these two approaches to analyse teacher's activity. In the following section of this paper, we will present the analysis of a case-study, based on the observation of ordinary teaching of algebra in French classrooms. In this study, Serge, an experienced teacher, devotes a lesson to introducing linear equations and setting up equations, in his class of 'Troisième' (14-15 years old pupils).

THEORETICAL FRAMEWORK FOR TEACHER'S ACTIVITY

A Model of Teacher's Activity Issued by the Theory of Didactic Situations

In the context of theory of didactic situations, Margolinas (2002) has proposed a model for the mathematics teacher's structuring milieu that allows to distinguish between different levels inside his activity and his knowledge:

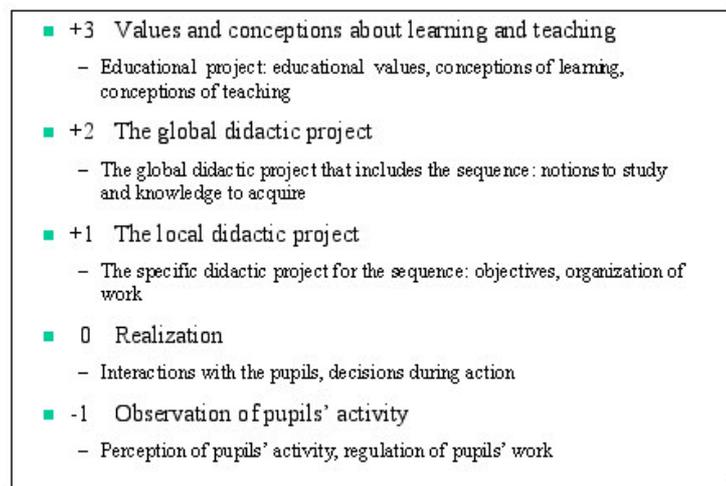


Figure 1:

levels of activity

Teacher's

This model takes into account the dynamic nature of teacher's activity by considering interactions at every level, with the upper components and the lower components (Perrin-Glorian 1999; Margolinas 2002). For instance, when the teacher is interacting with the students in a didactic situation (level 0), he deals with his local didactic project (level +1), and has to consider what he¹ observes of his pupils' activity (level -1). Furthermore, when the teacher prepares a mathematics lesson (level +1), he is considered in 'tension' between what he has planned for the sequence (level +2), and the interactions with pupils that he anticipates (level 0).

Teacher Knowledge and Practices in Social Institutions

In order to tackle teachers' various knowledge, we use the anthropological didactic approach. According to the theoretical framework developed by Chevallard (1999), the mathematics teacher's didactical, pedagogical knowledge is related to what is

¹ The masculine pronoun 'he' will be used, rather the compound 'he or she', to alleviate the text.

produced, practiced and diffused in social institutions (and particularly school institutions: official curriculum, textbooks, educational system, etc.). From this viewpoint, teacher's knowledge can be outlined by connecting with mathematical, didactical or pedagogical 'praxeologies', which are current in various institutions. In this way, institutional practices are seen as fundamental determinants which impinge on teacher's activity.

A CASE-STUDY: AN ORDINARY LESSON OF ALGEBRA

This case-study is based on my doctoral research (Coulange 2000, 2001), centred on the analysis of teacher's activity and teaching of systems of equations in an ordinary class of 'Troisième' (9th grade: 14 to 15 years-old pupils). The teacher, Serge, who is being observed, is an experienced teacher who has worked with researchers in mathematical education for a long time.²

The double theoretical viewpoint we propose leads us to construct a large data base with information not only about class interactions but also about the school institution, particularly for what concerns the current or past teaching of algebra. Our analysis is based on the complex data system, represented in figure2:

Classroom data		School institution data
Inside data	Outside data	Since the beginning of 20th to 1998
Observation of 3 ninety-minute classroom sessions on equations systems; audio recording of Serge and his pupils.	- Serge's interview before the lessons: his teaching project, lesson planning on systems of equations and the setting up of equations. - Written preparations - Serge's interviews after, during, between the observed sessions: his opinion about previous sessions and about changes in his project	Curriculum, mathematical syllabuses, textbooks, papers in didactic journals on systems of equations, the setting of equations, and on concrete problems: arithmetic and algebra.

Figure 2: Data system – Serge's teaching of systems and setting up equations

The algebra teaching project of serge

In order to address his global project of algebra teaching (level +2), Serge compares two didactical plans to introduce systems of equations. He calls the first one 'classical'. According to his comments during the interviews, the classical plan starts the lesson with the theoretical observation: «an equation including two unknowns may have an endless number of solutions». Serge claims that such an introduction of the equations systems allows to quickly introduce graphical solving of linear equations and sets of solutions. This 'classical' introduction seems to be inspired by

² He has worked in an IREM (Institute for Research in Mathematical Education).

past teaching practices: usually observed in 1970's and 1980's french school institutions.

But Serge criticised this 'obsolete' way of teaching. His actual plan is to confront algebraic and non algebraic strategies by submitting word-problems to his pupils. This confrontation is supposed to give meaning to the algebraic tools (systems of equations) as a better way to solve problems. This conception of teaching concerning the meaning of mathematical knowledge (level +3), is consistent with contemporary mathematics teaching ideology (stressed by papers in didactic journals, schoolbooks, etc.). The opposition between algebraic and non-algebraic strategies for solving concrete problems as a way to introduce linear equations (level +2) also seems consistent with contemporary teaching practices: it appears to be a major way to teach algebra contents in textbooks, or through an inquiry of teachers (Coulange 2001). What we call here 'non-algebraic strategies' represents numerical strategies that are not taught at secondary level. Secondary teachers often refer to these methods as 'trial and errors' strategies. Serge calls them 'arithmetical strategies', in accordance with his didactic knowledge.

Thus, his didactic project is based on nine concrete problems, taken from 'Petit x', a journal for mathematics teachers published by the IREM of Grenoble. We present here the first three problems:

1 Here are two heaps of stones.

x indicates the number of stones in the 1st heap.

y indicates the number of stones in the 2nd heap.

The second heap has 19 more stones than the first one.

a) Write an expression for y using x .

b) There are 133 stones in altogether. Write an equation defined in terms of x and y .

c) Find x and y .

2 Same problem as 1 with the following data:

- The second heap has 7 times as many stones as the first one;

- There are 56 stones in altogether.

3 Same problem as 1 with the following data:

- The second heap has 26 stones less than the first one.

- There are 88 stones in altogether.

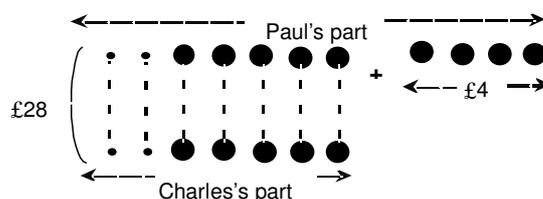
But what is the possible contemporary teacher's knowledge about arithmetical ways of solving the problems chosen by Serge?

In the French mathematics curriculum, the arithmetics' way of solving concrete problems was systematically taught in the first part of the 20th century, but it disappeared at the end of the 1960's. The 'stone' problem falls mostly into the 'unequal share problems' category. For instance, here is the translation of an extract from a 1932 french textbook (Delfault et Millet 1932).

Unequal share – 2 parts – sum and difference known

279. Typical problem – Paul and Charles share £28; Paul has £4 more than Charles. Find each part.

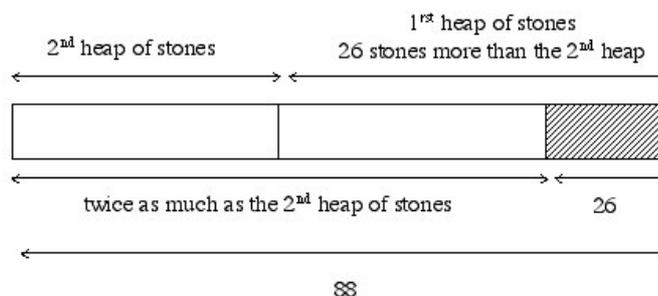
1st Solution. If I take £4 from Paul's part, I obtain Charles'. But the sum of the parts is reduced by these £4, making $28 - 4 = £24$ which is twice as much as Charles' part. Charles' part: $24 : 2 = £12$. Paul's part: $12 + 4 = £16$.



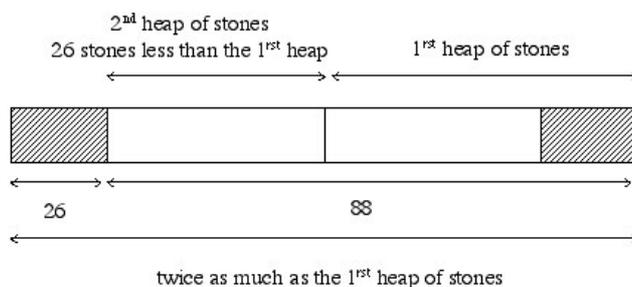
2nd Solution. By adding £4 to Charles' part, I obtain Paul's, the sum of the parts is increased by these £4, therefore it is equal to $28 + £4 = 32£$ which is twice as much as Paul's part.

Therefore, an arithmetical solution of the third 'stones' problem could be one of the following:

Solution 1. If I take 26 stones from the first heap, I obtain the number of stones in the second heap. But the total number of stones is reduced by these 26 stones, making $88 - 26 = 62$ and is twice as much as the number of stones in the second heap. The number of stones in the 2nd heap: $62 : 2 = 31$ stones. The number of stones in the 1st heap: $31 + 26 = 57$ stones.



Solution 2. By adding 26 stones to the second heap, I obtain the number of stones in the first heap. But the total number of stones will be increased by these 26 stones. Therefore it is equal to $88 + 26 = 114$ and is twice as much as the number of stones in the first heap. The number of stones in the 1st heap: $114 : 2 = 57$ stones. The number of stones in the 2nd heap: $57 - 26 = 31$ stones.



During the interviews, Serge talked in detail about algebraic strategies (setting up in equations, solving equations by substitution or addition), and it's possible to link his comments with specific algebraic knowledge. On the contrary, Serge's comments about arithmetical strategies were few and imprecise. And the disappearance of these arithmetic methods through the contemporary institutional context leads us to make the assumption that they are unknown to the majority of teachers. What does the teacher know about arithmetic learning and about 'stones problems' arithmetic

solutions? How may this subject-matter knowledge interfere with his didactic plans in progress?

Two Episodes in Serge's Class

During the observed lesson, as planned by Serge, his students produce non-algebraic solutions in response to the first 'stones problems'. But some of these solutions seem to be unexpected and unknown to the teacher.

Serge-Thibault episode

The following episode highlights Serge's lack of arithmetical knowledge.

In answer to the 3rd 'stone' problem, Thibault wrote:

$$\begin{aligned} x + y &= 88 \\ y &= x - 26 \\ 88 + 26 &= 114 \\ \underline{114} & \\ 2 &= x \\ x = 57 \quad y &= 57 - 26 \\ &= 31 \end{aligned}$$

Serge: why you added 26?

Thibault: this 26/here/first I did 88 plus 26/and I found it/I did 88 plus 26(...)

Serge: so/how did you get that/here you have 88 in altogether/here you say 26

Thibault: because x is minus 26/y is x minus 26

Serge: why are you doing/wait/no here it's correct/here

Thibault: these are just the terms of the problem

Serge: yes/OK/therefore 88

Thibault: I add on 26/that gives me 114/so then I divide by 2 to find the two heaps/in fact it's as if the two heaps were the size of x/I did it as if the two heaps were of the size of x

Serge: wait/let me have a look

Thibault: it's like there were two big heaps/therefore we have to add more on the right

Serge: Ah OK you reverse/therefore you add the difference Thibault: that's it

Serge: that is you add the 26 Thibault: that's it

Serge: and after you divide by 2/is that it

Thibault: I divide by 2 and take off 26

The first two lines of Thibault's solution are similar to the beginning of an algebraic solution. But they are only Thibault's answer to: "Write an expression of y using x". The lines that follow are similar to the second arithmetical solution (see above). Serge is at first totally confused by Thibault's solution, which gives the correct

numerical answer: he is unable to give the explanation required by the pupil. Apart from Serge's difficulty in understanding arithmetical strategies, this episode also reveals that even if arithmetical strategies are no longer taught, the students seem able to construct this reasoning for themselves in this situation.

Using the model of the structuring milieu, we can say that Serge is stretched with the upper components of the milieu, that include the contemporary establishment's relationship to algebra and arithmetic and the absence of arithmetics as a body of knowledge within the secondary programs. Serge is unsettled by the student's ability to create sophisticated arithmetical reasoning, instead of mere 'trial and error' strategies. In fact, Thibault's explanations offer Serge an opportunity for learning. Immediately after the Serge-Thibault episode, Serge rushes to inform an observer, present in the classroom but who has not observed the interaction.

Serge: hey this is a good one [to Observer 1 beside him] take a look at this/wait I want to show it to Observer 2 [further away] [to Observer 2] he reverses that is he adds 26/that is he adds on the two heaps at maximum/therefore at the beginning 88 / hence he adds on 26 in order to have two equivalent heaps and he divides by 2 [...] and therefore he has the big heap/afterwards he takes off 26

In this interaction, Serge shows the observer that he now fully understands Thibault's arithmetical strategy, even if a few moments before, he wasn't able to answer Thibault's question. There has been an evolution in Serge's knowledge of this arithmetical solution: Serge learned during the chosen episode. During his interaction with Thibault, Serge was clearly dealing with an antagonistic milieu, which contains Thibault arithmetical reasoning (level -1). The conditions for the existence of this antagonistic milieu are linked to the upper components of the milieu: he pays attention to the mathematical activity of his pupils (level +3: values), he particularly observes emergence of non algebraic solutions (levels + 2 and +1 of global and local didactic project). Nevertheless, the contemporary mathematical teaching institution doesn't consider arithmetic as a body of mathematical knowledge that is suitable to teach. We suppose that the learning of such arithmetic knowledge is not really supported by his local or global didactic project, in conformity with the institutional project for teaching algebra. Therefore we can foresee that Serge is in no condition to learn in any stable way, from his interactions with Thibault (Margolinas et al. 2004).

Serge-Benoît episode

Nearly at the end of the session, Serge requests Benoît to go up to the blackboard and to write down his solution to the eighth 'stone problem':

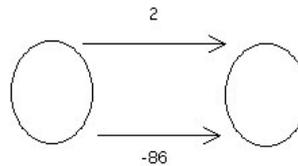
8.—We know that: The first heap has twice as many stones as the second; the third heap has 36 stones more than the second; the second has 86 stones less than the first. Find x , y and z .

Considering only the information given about the first and the second heaps of stones, we can stress a problem which falls into another 'unequal share' category:

knowing ratio and difference. Following Serge's didactic plan (level +1), at this time, the opposition between algebraic and non-algebraic strategies is supposed to have worked in the previous way: the teacher expects only algebraic solutions from his pupils. To his surprise, Benoît launches into explanations of an arithmetic solution:

Benoît: the heap has twice stones more than the second / and at third, we consider that the second heap has 86 stones less than the first / hence I used 86 as a point of reference for the second heap / because the other has twice more / and

(Benoît draws down on the blackboard)



Serge: try to

Benoît: the additional stones...

Serge: wait / it would be easier if you drew 3 heaps and you explain with the drawing

Benoît (adding a heap on his sketch): 1 heap, 2 heaps, 3 heaps, the first heap is equivalent to twice as many stones as the second, okay / the second one has 86 stones less than...

Serge: be careful / it is the second heap / the second has 86 less than the first

Benoît: therefore I used 86 as a point of reference for this one / and I made two times more / I multiply 86 by 2, it gave me the first heap, and it made 86 less / the first heap really had twice as many as the second and the second heap actually had 86 less than the first

Serge: therefore, you find the third / Hence you take into account the two first heaps / wait, wait / (...) / yes / once you have the two first ones, it is correct...

Benoît: the first / so it makes

Serge: right / once you had the two first and you start from 86

Benoît: as a point of reference

Serge: I am not completely sure / we'll see if your trick will always work

Serge seems to be completely confused by Benoît's solution. Once more, this episode reveals the teacher's difficulty in understanding arithmetical strategies: at first, the teacher doesn't understand the reasons why Benoît takes into account only informations given on the two first heaps. Nevertheless, on the contrary to Serge-Thibault episode, this interaction doesn't offer the teacher a real opportunity for learning arithmetical knowledge. Benoît's solution is quite unexpected for the teacher: Serge doesn't observe this pupil's activity (level -1) and plans the disappearance of arithmetical strategies at this point of the lesson (level +1). Furthermore, Benoît's public intervention may create a disturbance in Serge's project of teaching algebraic knowledge (level 0, +1). Consequently, according to the upper components of the milieu, the teacher decides to fudge the pupil's explanations.

Actually, the opposition between algebraic and arithmetical strategies planned by the teacher doesn't work this way: some pupils don't give up their arithmetical strategies. This situation leads Serge to clarify his expectations concerning algebra contents, and to leave less and less room for students to embark on non-algebraic strategies. During the session, we observed how Serge managed his didactical plan of teaching algebra, by using the mixed group of the class (Coulange 2001).

To conclude on these two episodes observed in Serge's classroom, we stress that as many contemporary teachers, Serge seems to be rather ignorant of arithmetical categories of problems and solutions. This 'institutional ignorance' constrains strongly the management of his didactical project: Serge is led to fudge more and more 'public' interventions related to arithmetical strategies, and to clarify his expectations related to algebraic knowledge, in order to manage his teaching of algebra. We can notice that the teacher's situation sometimes enables him to become aware of the pupil's arithmetical strategies that he has not anticipated. For example, the Thibault-Serge episode offers Serge an actual opportunity to learn arithmetical contents. But the understanding of arithmetic knowledge is not supported by his local or global didactic project of algebra. Even if this teacher considers the mathematical attempts of the pupils potentially meaningful, he is unaware of his own ignorance about arithmetical solutions: the institutional context prevents him from considering arithmetic knowledge as useful for his teaching of mathematics.

CONCLUSIONS AND PERSPECTIVES

Through the use of a double theoretical framework inspired by the theory of didactic situations the anthropological approach, we were able to examine the different determinants of the teacher's situation. In this case-study, the study of institutional teaching of algebra, and arithmetic knowledge highlighted Serge's didactical projects, interactions with pupils, and the way he managed the class. Also, the model of the *milieu* structuring allowed us to study Serge's didactical knowledge way of working and the reasons behind his decisions during the session observed.

Furthermore, in this paper, we outlined the importance of didactical knowledge in mathematics teacher activity and the influence of various institutional determinants upon this knowledge. By briefly addressing relations with the twofold approach developed by Robert and Rogalski (2002), we suggest that an in-depth study of institutional contexts allows to tackle an important part of a teacher's 'personal history', knowledge and beliefs about mathematics.

To conclude, this study leads us to question the issues of our findings for pre-service and in-service training. At this time, we reach two main conclusions:

- It is necessary to take into account mathematics teaching practices, current in past and present institutions, in order to convey training contents, and to better understand a teacher's actual knowledge.
- Enabling mathematics teachers to think about their practices requires external intervention which could give them insight into their interactions with pupils. Even if

teachers learn on their own from experience, the stability of such knowledge rests both on institutional determinants and on training experiences.

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LEARNING WITHIN COMMUNITIES OF PRACTICE IN PRESERVICE SECONDARY SCHOOL TEACHERS EDUCATION

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Abstract: *We present the main features of a study that explored the learning processes of a group of preservice mathematics teachers in a methods course. We discuss the implications of using the communities of practice perspective in designing and developing teacher training programs.*

Keywords: Teacher learning, Teacher education, Communities of practice, Wenger, Preservice, Secondary Mathematics.

Learning in preservice mathematics teacher education

Teachers neither work, nor learn alone. Teaching and learning to teach are social practices and collaborative enterprises (Secada & Adajian, 1997). That is why research in teacher education has become increasingly concerned with teachers' development from perspectives rooted in sociocultural views of learning (Lerman, 2001; Llinares, 1998). In particular, Wenger's social theory of learning (Wenger, 1998) and its notion of community of practice are becoming popular as a conceptual framework for exploring the learning processes of mathematics teachers working together. However, "while mathematics teacher education researchers are creating contexts that enable teacher learning and describe what teachers learn in social terms, little has been done to explain *how* those contexts enable learning" (Graven & Lerman, 2003, p. 189). Furthermore, there has been little research examining the specific interactions and dynamics that happen in those contexts: "One analytic task, therefore, is to show how teachers, in and through their interactions with one another and with the material environment, convey and construct particular representations of practice" (Little, 2002, p. 934). As Krainer (2003) has pointed out, there is much to be explored concerning the role of this perspective in teacher education: "To what extent can an approach like 'Community of practice' be applied to learning at schools and universities? What can we learn from 'learning enterprises'? What implication for research in teacher education has an approach that builds on 'community of practice'?" (p. 96).

Within the research agenda described by the previous questions, we present and discuss the main features of a study that, in the context of a methods course, explored the learning processes of a group of mathematics preservice teachers working at home. For that purpose, we first describe the methods course, introduce Wenger's social theory of learning, and portray the methodology used. Then, we present an example of the results obtained, and we argue that the group developed a community of

practice. Finally, we discuss the implications of using the community of practice perspective in the design and development of teacher training programs.

Preservice teachers' learning in a methods course

The study was part of a research project exploring the didactical knowledge development of secondary preservice teachers in a methods course. Preservice teachers were asked to form groups of four to six persons at the beginning of the course. Each group chose a Secondary School Mathematics topic on which it worked during most of the course. Examples of topics were the quadratic function, sphere, the Pythagorean theorem, and decimal numbers. During the second part of the course (in which the study was conducted) the groups of preservice teachers were asked to sequentially analyze and describe their topic from different points of view. For example, they had first to produce the conceptual structure of their topic, then identify the representation systems that can be used to represent it, and then to perform a phenomenological analysis of it. The groups worked at home solving the tasks. They produced transparencies with the help of which they presented their work to the classroom. In other studies of this research project we explored the results of the work produced by the groups of preservice teachers. These studies allowed us to identify and characterize four stages of didactical knowledge development with which to describe the groups of teachers' performance over time (Gómez & Rico, 2004). However, a question remained concerning how this didactical knowledge developed in the groups. Those studies analyzed the results of learning processes performed by each of the groups. But what kind of processes were there? Is it possible to describe and characterize some of those processes? In other words, is it possible to explore the emergence of learning that took place in a group? Can this exploration help explain the didactical knowledge development of that group? For this purpose, we had chosen one of the groups of preservice teachers and had asked their members to allow us to record in audio their group interaction while preparing their presentations for the course. This group had the quadratic function as its topic of study. Eight meetings were recorded, producing 18 hours of recording.

We had then to approach a theoretical and methodological problem. How to explore, describe and characterize the learning of a group? Following Stein and Brown (1997) we decided that “rather than focusing on the learning processes of individual teachers undergoing transformation, [we could conceptualize] teacher learning as a process of ‘transformation of participation’ in the practices of a community” (p. 155). For that purpose, we decided to ground our study on Wenger’s social theory of learning (Wenger, 1998).

Learning as a social practice

Wenger’s social theory of learning is based on four notions: meaning, practice, community and identity. He introduces *meaning* as a way of taking about our (changing) ability —individually and collectively— to experience our life and the world as meaningful. The negotiation of meaning emerges from the interaction of two proc-

esses: *participation*, as the process in which we establish relationships with other people, we define our way to belong to the communities in which we engage on some enterprises, and we develop our identity; *reification*, as the process of giving form to our experience by producing objects that congeal this experience into “thingness”. Every community produces its abstractions, tools, symbols, stories, terms and concepts that reify some of the practice in congealed form. The notion of *practice* is presented as a way of talking about the shared historical and social resources, frameworks, and perspectives that can sustain mutual engagement in action. Practice is the source of coherence of the communities and the process through which we experience the world meaningfully. It does not exist in abstract; it exists because people engage in actions whose meanings are negotiated. The idea of *community of practice* represents the smallest unit of analysis in which one can include the negotiation of meaning as mechanism of learning. It is a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognizable as competence. The idea of community of practice is configured on three notions: mutual engagement, joint enterprise and shared repertoire. Finally, the notion of *identity* is introduced as a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities. Learning as a social practice can be characterized by the three notions configuring the idea of community of practice: *learning in practice implies a mutual engagement in the search of a joint enterprise with a shared repertoire*. That is, learning in practice implies:

- ◆ *Evolving forms of mutual engagement*: how to engage, what helps and what bothers, developing mutual relations, defining identities, establishing who is who, who is good at what, who knows what.
- ◆ *Understanding and tuning the enterprise*: aligning the engagement, accountability and responsibilities, defining and interpreting the enterprise.
- ◆ *Developing the repertoire*: renegotiating meanings, producing and adopting tools, artefacts and representations, recording and recalling events, inventing and redefining terms, telling stories, creating and breaking routines.

Learning as a social practice in preservice teacher training

Figure 1 presents a diagram of the methodological procedure that we used to operationalize Wenger’s social theory of learning in order to study the quadratic function group’s work. We decided to focus our attention on the three processes implied by learning in a community of practice: evolving forms of mutual engagement, understanding and tuning the enterprise, and developing the shared repertoire. These are the three *categories* of analysis. Based on the description that the theory makes of these processes and the specificity of our research context, we characterized each category in *aspects* and for each aspect we produced a set of questions, that we called *themes*. For example, the mutual engagement category was organized in four aspects: the role of the environment (what helps, what bothers), the building of identities, the

evolution of relationships among the members, and the processes of negotiation of meaning. Concerning the processes of negotiation of meaning, we identified five themes: the production of meaning proposals, the adoption of meaning proposals, meaning difficulties, discovery of meaning and reification. In the case of the theme meaning difficulties we produced two *codes*: events of confusion of meaning, and events of conflict of meaning. On the basis of this procedure for interpreting the theory and contextualizing it to a research context, we produced a first version of the codes set. This codes set was revised during a first partial coding of the transcriptions. The result was a set of 94 codes. For example, we assigned the code *teaching experience* to those episodes in which at least one participant refers to his experience as a teacher.

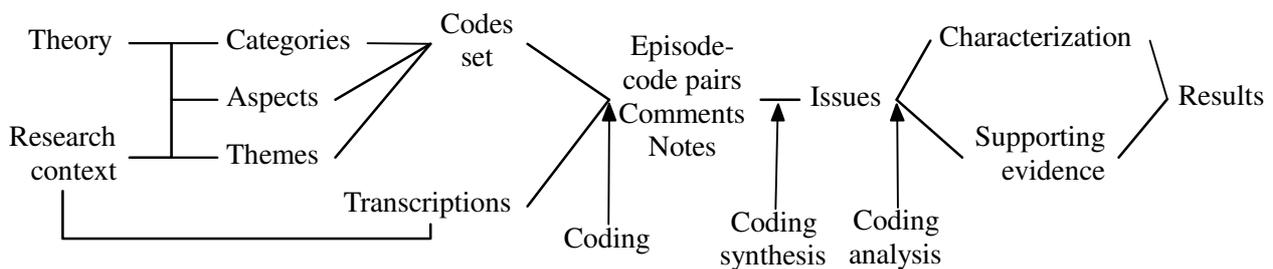


Figure 1. Operationalizing Wenger's social theory of learning

We coded the transcriptions, producing 7,412 episode—code pairs corresponding to 2,606 episodes (since several codes could be assigned to a given episode). For each episode, we produced a comment, in which we described what the content of the discussion in that episode was about. Furthermore, parallel to the coding process, we produced a series of notes in which we registered aspects of the interaction that either could not be characterized by the codes or went farther than what the coding allowed us to register. This coding process produced a huge amount of very detailed information. Through a process of *coding synthesis* we identified the main *issues* concerning the learning of the group that appeared in the coded transcriptions. Finally, through a process of *coding analysis* we were able to establish the main characteristics of each issue and identify the episodes that were more representative of each characteristic. Our approach was similar to the one used by Little (2002). We also shared with her work the purpose of such a fine-grained procedure: “to produce well-grounded assertions regarding social practice and learning” (p. 920).

Negotiation of meaning: confusion

We present here a glimpse of one of the results of the study. It concerns confusion of meaning. This is one of the several issues we found in relation to negotiation of meaning in the group. We characterized the episodes of confusion of meaning as those in which, for a particular question, one or more of the participants: are not sure about its meaning, change their opinions about its meaning across the meetings, or make meaning proposals that are not valid.

The difference between the notions of equation and function was in the centre of a confusion of meaning that spanned during several meetings. While working on the

history of their topic, the group got interested in the relation between those two notions and some of its members engaged themselves in a historical exploration, with the hope of solving the issue. However, the confusion reappeared while working on the mistakes and difficulties that the students in school might have with their topic. The confusion remained for some time. However, in the last meeting we found evidence showing that it had been resolved.

The following episode belongs to the meeting in which the group worked on the history of their topic. One of the participants thinks that he knows the difference between the notions of equation and function. However, the confusion appears when the group tries to establish such a difference. They start by stating that the quadratic function is the generalization of the quadratic equation and they finish with an emphatic claim: every quadratic equation is a function. This meaning is adopted by the group and is reified in the transparency produced in this meeting [043, 8000, 10438]¹:

P3: What I mean... I am going to say it: we know very clearly what an equation and a function are, because... What can I tell you... Perhaps because for the last 30 years a difference has been established, and that is what we have been taught.

P2: OK, but you are not going to explain that.

P3: No, no. I agree. Wait, what I want to say is... I am not talking about us. Why all this mess? Because we think in a certain way. Equation and function. That is: function, when is the term function used? When you have to give...

P2: A relation between variables, some magnitudes...

P3: A relation between one variable and another, between a magnitude and another. But the equation was there since the beginning. And equation of second degree, it is simply a question of the change of one thing with respect to the other; with the equation of second degree. That is a function of second degree. Therefore, we are going to talk about equations of second degree, and then we tell them...

(...)

PX: What happens is that for me, the generalization of an equation of second degree is in fact a function.

P3: OK, it is a function. It can also be that.

PX: No, it is not that it can be, it is.

P3: OK, it is.

PX: And what happens is that any equation of second degree is a function.

PX: But since this is a work on history...

PX: Yes (several participants talk simultaneously).

¹ Numbers in square brackets identify the location of the corresponding episode in the coded transcriptions.

In this episode, we can see some aspects of a process of negotiation of meaning within the group. Firstly, there is confusion concerning the meaning of the notions of equation and function, and the difference between them. This confusion is made explicit due to the fact that they are looking at the history of their topic. But there is also confusion because the group adopts a meaning of the two notions that is not valid with respect to established mathematical knowledge: “the generalization of an equation is in fact a function”. Secondly, we see participation. At least three of the four members of the group participate in the discussion and make comments and proposals. The members of the group have become used to ask questions and to expect reactions from the other participants: “when is the term function used?” This can be interpreted as one of the ways through which members care for each other’s learning. Thirdly, there is conflict. One of the participants, PX, has an idea and puts it forward. Another participant, P3, interprets this proposal: “OK, it is a function. It can also be that”. But this was not the meaning proposed originally by PX. He emphatically corrects this interpretation, without further arguments: “No, it is not that it can be, it is”. This is one of the mechanisms of conflict resolution: a proposal without arguments that is accepted by the group. Fourthly, a member makes a reference to their mathematical preparation in their career. Finally, there is reification. The group adopted this proposal and it was reified in the transparency that they presented to the class.

The emergence of a community of practice

The above example of an episode gives a glimpse of the behaviour of the group as a community of practice. In particular, it shows instances of some of the features of their processes of negotiation of meaning. However, confusion of meaning is only one of the thirty relevant issues we identified and characterized. The structuring of those issues and the evidence supporting them enabled us to produce an account of the working of the group as a community of practice. We do not have space here to present such an account. Next we describe, as an example, some of those issues.

We found that one of the participants had become the leader of the group. We characterized his role as a leader in terms of his forms of participation as well as describing the forms of “complementary participation” shown by others members in relation to the leader’s behaviour. On the other hand, we found three elements related to the group as a community of practice that had clear and specific effects on the processes of negotiation of meaning within the community. First, most of the members had some teaching experience. As matter of fact it is clear that the leader was accepted as such because he had what was seen as the most thorough of teaching experiences. But all members recall their teaching experience and construct stories based on it in order to make proposals of meaning, and to put forward arguments supporting those proposals. Second, even though the trainers gave a list bibliographic references concerning mathematics education literature, the group did not mention any of those references. On the other hand, textbooks played an important role in their discussions. They used information in the textbooks for resolving some of their confusions and conflicts of meaning. Furthermore, their use of textbooks was central in the design of

the activities that shaped their proposal for a didactical unit. Finally, one of the trainers handed in written commentaries to the presentations made and transparencies produced by each group. Even though the group did not take into account these commentaries in every meeting, they were a significant factor in the group's working, while producing the final document. It was at this moment when, by reviewing the commentaries, the participants were able to resolve some of the most resilient confusions and conflicts. Although there are references in the transcriptions to what happened in the classroom (interaction with peers, spoken commentaries from the trainers to their presentations, or general statements of the trainers), we found that the commentaries to the transparencies were the most important link between the working of the group and the classroom environment. In Wenger's terminology this was the most important *boundary object* (Wenger, 1998, p. 104) between the group as a community of practice and the community of practice of the classroom.

In the episode we presented above we saw that participants cared for each other's opinions and expected arguments supporting them. This type of participation promoted *interdependent learning*, one of the most important features of a community of practice. This process of search of meaning generated events of confusion, conflict and discovery. We characterized the mechanisms used for these types of events. Given that the group had to solve a task in each meeting, these processes of negotiation of meaning always ended in the adoption of some proposals that were reified and registered in the transparencies. These were some of the objects of reification of the group and served as reference for their discussions later on.

Communities of practice: a conceptual tool to see, think and act²

Wenger's social theory of learning enabled us to "see" through the complexity of 18 hours of recordings. On the basis of some aspects of this theory, we were able to systematically construct conceptual categories and design instruments for coding and analysing the information in such a way that we could, at the same time, explore the data in detail, and synthesize and analyze the results of that exploration. We identified and characterized a series of issues that give an account of learning as a social practice in the group.

A tool to "think"

We are not suggesting that all groups established themselves as communities of practice or developed learning processes as those depicted by the group we studied. For example, we have evidence of groups in which a leader did not emerge. On the other hand, the analysis of the classroom interaction and of the final documents that we performed in another study suggests that some groups organized themselves as teams: the tasks were divided into subtasks, each member taking responsibility for delivering his/her part. The presentation is then built as the summing of the parts. As Anderson and Speck (1998) have shown, when a group organizes itself as a team,

² "A theoretical discourse is not an abstraction. It is a set of conceptual tools that enable us to see, think, and act in new ways" (Wenger, 2004, p. 2).

there is learning. However, one cannot expect negotiation of meaning and interdependent learning as properties of the learning processes of a team. What was important in the case of the quadratic function group we studied was the mutual engagement of the participants in the search of a joint enterprise that involved the concern for the learning of all the members of the group. It is not a question of collaboration *versus* cooperation (see, for example, Beck & Kosnik, 2001; Peter-Koop et al., 2003). We think that, in contrast with the community of practice perspective, the collaboration approach neither takes into account the complexity of the interactions, nor proposes an overall conceptual structure for describing and analyzing the learning processes involving negotiation of meaning.

The working of the groups is *one* of the contexts in which learning takes place in our methods course. Preservice teachers, individually and collectively, also learn, during the lessons, while doing individual work, in other courses, and while giving private lessons. However, given that the formal assessment that we do of our students gives a high relevance to the presentations of the groups and the documents submitted by them, and that those presentations and documents are produced as a result of the working of the groups, it is now clear to us that we have been specially valuing the learning processes that take place while the groups work at home. Since teaching takes place during the lessons, one tends naturally to think that most learning happens within that context. This study has shown us that this is not necessarily the case.

A tool to “act”

The previous paragraphs show how the notion of community of practice has allowed us to “think” about some aspects of our methods course. These thoughts suggest some ideas about how to “act” in the future. Wenger (1998) mentions the risks of romanticizing communities of practice (p. 132). Nevertheless, research on teacher education is giving increasing importance to communities of practice as prominent loci of learning and development in teacher training. For instance, communities of practice can enhance the learning capability of preservice teachers (Knight, 2002, p. 240; Wood & Berry, 2003, p. 65), develop the awareness of the value of collaboration (Beck & Kosnik, 2001, p. 925), help counterbalance the long apprenticeship preservice teachers have had in transmission pedagogy (p. 945), and encourage the building of professional communities in the future (Lachance & Confrey, 2003, p. 38). If we, as trainers, value the learning that takes place when a group works as a community of practice, how to promote and cultivate such a setting? Answering this question requires that teachers’ trainers, besides taking care of *what* they expect preservice teachers to be able to do and to know, get concerned about *how* preservice teachers learn and *what* kind of *teaching* is coherent with that learning. The design of the training program (in particular, issues as the methodology and the trainers’ performance and attitudes) can make a difference in that learning. Next, we consider three of those issues. They are examples of the type of questions that should be considered while designing a methods course in which communities of practice and interdepend-

ent learning are expected to take place. They refer to the trainers' written commentaries to the groups' work, the definition of the tasks, and the groups' tutoring.

One of the most relevant issues emerging from our study was the characterization of the role that the commentaries to the transparencies played in the processes of negotiation of meaning of the group. The commentaries to the transparencies emerged as the main reference to the socially defined competence of the classroom community. Instead of giving the solutions to the problems observed, the commentaries proposed new questions and opened new spaces for discussion and reflection. In this sense, the commentaries to the transparencies promoted and guided new processes of negotiation of meaning and enhanced interdependent learning within the group.

Another aspect of the design of the course that had an influence on the processes of negotiation of meaning of the groups was the way the tasks were defined and proposed. Preservice teachers expected clearly defined tasks in which they "knew what they were expected to do". However, the tasks usually proposed a general problem (the analysis of the group's topic —e.g., quadratic function— with a given conceptual tool —e.g., materials and resources—) that each group had to contextualize and solve according to their own topic, their previous knowledge and experience, the information they could collect and the shared repertoire that they had developed in the previous meetings. The tasks were proposed in such a way that there was always a challenge involved, but solving them was not seen as impossible by the groups. In this sense, the tasks promoted interdependent learning.

The design of the tasks and the commentaries to the work produced by the groups might promote interdependent learning in a group *if they have already constituted a community of practice*. Otherwise, in a group working like a team, the commentaries to the transparencies and the definition of the tasks are usually interpreted within the working routines already established and do not necessarily promote the negotiation of meaning. If we value the learning that takes place when a group works as a community of practice, how to promote and cultivate such a setting? Answering this question would require, in our case, a change of attitude as teachers' trainers. While interacting with the groups (in the classroom or in tutoring meetings) we focused our attention on *what* the groups had learned and tried to help them in improving their work (transparencies, presentations and documents). However, we know now that we have also to take into account the *learning processes* that give rise to the productions of the groups and to look for ways of promoting interdependent learning and negotiation of meaning in the groups.

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DEVELOPING MATHEMATICS TEACHER'S COMPETENCE

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Abstract: *The attention paid to the teacher's competence as one of the determining factors of the quality of the teaching of mathematics systematically grows. Considering the complexity of the teacher's task, it is necessary to see teachers' competence as a structure that consists of, among others, subject-didactic and pedagogical competence. The question, whether any aspect of teacher competence plays a dominant role, appeared. The paper is based on the comparison of approaches of two teachers to the elaboration and realization of the instruction experiment dealing with the creation of preconceptions of fractions.*

Keywords: mathematics teacher's competence, teachers' knowledge base, primary school level, fractions

INTRODUCTORY REMARKS

The concept of competence is among those that have been the focus of pedagogical and didactic research in the Czech Republic. Competence is mentioned in connection with the professionalisation of the teacher's knowledge and defining "mathematics" and "teaching" as essential components of mathematics teacher's activity (Scherer & Steinbring, 2003). We have been investigating the characterisation of elementary mathematics teachers' competence and the possibilities of their development for a long time. We have paid attention to this problem within an international project of the Socrates Comenius programme, too (Hospesová and Tichá, 2003a). We reported it in our contribution published in the proceedings of CERME-3. This article links up with that contribution.

TEACHER'S COMPETENCE

We utilize the term "teacher's competence"¹ to denote a set of professional skills and dispositions that the teacher should possess and be ready to develop in order to carry out his/her job effectively. We emphasize its dynamic conception (Spilkova, 2001). A teacher's practice is characterised by great complexity, therefore, the teacher must have and continually improve an arsenal of competence to be able to react to the situations that arise in the classroom and reflect on them. We understand teachers' professional competence as a complex qualification for a successful performance of

¹ We used the word "competence" because in our opinion it is broader than the term "knowledge". Among other things, it stresses the ability to act.

the profession, which (a) is based on a theoretical reflection on practical experience and (b) includes knowledge of subject, skills, attitudes, experience, values and personal characteristics.

Our understanding of the notion of teacher's competence is strongly influenced by the work of the Czech psychologist Z. Helus, who emphasizes that "A successful effort to change the school is only possible if the teacher becomes its leading agent. It is to do with answering questions regarding his/her competence and responsibility, regarding his/her appropriate good condition, regarding updating his/her pre-service preparation and providing lifelong education. This implies a change in the demands on the teacher's knowledge and competence." (Helus, 2001). Helus also points out that the basis of the teacher's self-confidence includes the following competence (for more detail see Hospesová and Tichá, 2003a):

- (a) Pedagogical competence consisting of (1) creating conditions (climate), (2) removing mental blocks and barriers, (3) mastering diagnostic operations, (4) getting an insight and empathy, and (5) designing procedures for pedagogical intervention.
- (b) Subject-didactic competence consisting of a skilled orientation towards the educational meaning of teaching a specific subject, mastering the scientific basis of the subject (mathematics), as well as didactic creativity.
- (c) Pedagogical-organisational competence.
- (d) Competence in a qualified pedagogical (self-) reflection.

Helus's specification of teacher competence covers all activities, which occur in instruction (including its evaluation by means of reflection). It corresponds to our notion about the complex nature of the teacher's profession and we take it as a basis of our considerations concerning teacher competence.

We would like to add that our characterization of teacher competence was also influenced by the authors who deal with "the knowledge" needed for performing the teacher's profession on an appropriate level and who also usually stress the mutual relation of school instruction and its theoretical background (theory and practice). E.g. Bromme (1994) distinguishes five fields of knowledge that are needed for teaching: (a) knowledge about mathematics as a discipline; (b) knowledge about school mathematics; (c) philosophy of school mathematics; (d) general pedagogical knowledge; (e) subject-matter-specific pedagogical knowledge. He considers (b) and (e) as the most important. Harel and Kien (2004) indicate three interrelated critical components defining teachers' knowledge base (a) knowledge of mathematics content (refers to the breadth and ... depth of the mathematics knowledge), (b) knowledge of student epistemology (teachers understanding of fundamental psychological principles of learning ...), (c) knowledge of pedagogy (refers to teachers' understanding of how to teach in accordance with these principles ...). Bromme, Harel and Kien (and many other authors) refer to Shulman's theory of pedagogical knowledge and his considerations about the knowledge base of a teaching profession (Shulman, 1986, 1987). According to Shulman, a teacher should have at his/her disposal content knowledge that includes "subject matter content

knowledge”, “pedagogical content knowledge” and “curriculum knowledge”. “Pedagogical content knowledge” is characteristic for the teaching profession, as together with the knowledge of content, it covers the knowledge of how to teach.

COMENIUS PROJECT AND CULTIVATION OF TEACHER’S COMPETENCE

Since 2001, we have been cooperating with German and Italian colleagues within the Comenius project “Understanding of mathematics classroom culture in different countries”. The aim of the project was to improve the quality of continuous in-service education of primary school teachers through changing the character of teachers’ work in the teaching of mathematics. (For more information about the project, see www.pf.jcu.cz/umccdc.) In the studies, which originated within the project, we have concentrated so far on the mutual influence between conducting qualified pedagogical reflection on the one side and improvement of the level of teachers’ competence on the other (Hospesová and Tichá, 2003a). We focused especially on subject-didactic and pedagogical competence.

When preparing, carrying out and mainly analysing instruction experiments, a question was raised whether any teacher competence is “superior” to the others to such an extent that its low level prevents the development of other competences. (Similar questions appear in different studies, for instance, in the introduction to WG3 at CERME2: What is the role of mathematical knowledge? How can we keep a good balance between subject knowledge and pedagogical knowledge?). Our question of the hierarchy of competence emerged in the analysis of the following instruction experiment.

CONTENT AND ORGANISATION OF AN INSTRUCTION EXPERIMENT

The instruction experiment under consideration was carried out within the above-mentioned Comenius project. Since the beginning, the key feature of our project has been the collaboration of primary school teachers, teachers from universities (involved mainly in pre-service teacher training) and researchers. During our work on the project, an intensive co-operation between teachers themselves and between the teachers and researchers has developed into several relatively stable stages. (a) In one of the meetings of the team, a topic was discussed which the teachers considered difficult from the point of view of its mathematical content, goals and possible didactic elaborations, of didactic difficulties, of students’ expected reactions, etc. (b) The teachers individually (according to their experience and knowledge of used teaching methods and procedures) prepared an instruction experiment, that (c) was videotaped and (d) the video recordings were reflected upon in a collective discussion of teachers and researchers. (Hospesová and Tichá, 2003a).

Whole/part relation and fractions

We consider laying the foundations of the understanding of the whole/part relationship (in particular in creation of imagery and development of understanding

of the notion of fraction) to be one of the basic topics of the teaching of mathematics at the elementary school. It is very demanding for both pupils and teachers. We often meet misconceptions and insufficient grasping of this topic in teachers themselves (Tichá, 2003). We therefore welcomed the fact that the cooperating teachers within the Comenius project decided to concentrate on this topic because they felt shortages in this field regarding their own knowledge of mathematical content and its didactic elaboration (Hospesová and Tichá, 2003b, Tichá and Hospesová, 2004).

We spent several team meetings looking into the development of the images of fractions and into the diagnosis of pupils' preconceptions. After carrying out and analysing several teaching experiments together, the teachers themselves were able to identify problems necessary to pursue and suggest an instruction experiment. We thought that the experiment suggested by the teachers themselves was the confirmation of the above-mentioned improvement of subject-didactic competence. However, the realization of the experiment and analyses of its videotape brought about great disillusion and raised many questions and problems.

Next, we will briefly describe the origin and realization of one instruction experiment prepared by the teachers themselves.

Starting point - collective discussion

Teacher A suggested an experiment aimed at the development of multiple representations: "To each pupil, I will give a stripe of paper divided into equal parts such as the one I have on the blackboard [Fig. 1] and also here in my hands (she showed it). I will ask them what they can see. I will observe if they start speaking about fractions when folding the paper. And also whether they will call the same part one half, two quarters, and four eights."



Fig. 1

Teacher B decided for the realization of the experiment in the proposed context, i.e. a stripe of paper, too. The proposal was discussed during a team meeting from various points of view. Then both teachers A and B prepared a lesson individually.

Different realization of the experiment

In view of the experience we had within the project (Hospesová and Tichá, 2003a), we supposed that there would be different realizations of the experiment. Both teachers followed the original suggestion, although only loosely. Teacher A realized this experiment with grade 4 (10 years old), teacher B with grade 5 (11 years old). All these pupils had met fractions during school lessons only marginally.

"Discovering fractions" in teacher A's class

Teacher A, who originally suggested the experiment, abandoned the original idea of multiple representations. She kept the context and tried to create such a situation in which "pupils will discover fractions independently and will be able to distinguish between a whole and its parts". She did not specify further what she meant by "pupils

independently discovering fractions”. When we asked her, she was not able formulate it clearly in more detail neither before lesson nor after its realisation. Therefore, based on the course of the lesson we can only guess that she wanted to focus on some properties of unit fractions in addition to whole/part relation.

Teacher A formulated two tasks. For Task 1, teacher A distributed stripes of paper divided into four equal parts by folding (see Fig. 2) and posed the task by saying: “We are discovering fractions. Please, look at what you have in front of you and try to express it somehow“



The pupils’ reactions were diverse, e.g.: “A whole divided into four parts“, “These are four connected rectangles“, “A line segment of four parts“. The teacher then asked them to write what they could see “in a mathematical way”. The pupils suggested various possibilities, three of which the teacher let them write on the blackboard:

$$1 \quad 4 \quad \frac{4}{4}.$$

The teacher completely ignored some students’ suggestions (without any argumentation), especially geometrical interpretations. Next, she asked the pupils to decide which record was correct. Students’ answers were different; next they voted correct answer. In this phase, teacher accepted only answer “1“ as correct (again without any argumentation probably because she wanted to stress “whole”, “unit”).

For Task 2, the pupils divided the stripe of paper into 4 separate parts (see Fig. 3). Again, the teacher asked them to write what they could see “in a mathematical way“.

Different proposals appeared: $\frac{1}{4}$, $\frac{1}{1}$, $\frac{4}{4}$, $\frac{4}{1}$, even $\frac{3}{4}$ (“3 spaces between 4 quadrangles”). It was obvious that the pupils hesitated what the whole is and that different ways of grasping of the situation are possible. The follow-up discussion revealed that the pupils tried to suggest ways that could be accepted by the teacher. (It was interesting that no geometric interpretation appeared this time.) Their strategy can be summarised as “to find the answer which the teacher would label correct“ and “argumentation is not needed”.



The teacher accepted answer “four quarters” and rejected the other proposals without explanation again. One pupil tried to explain this decision: “We know that we divided the rectangle into 4 parts. They are quarters. And we know that they are 4. So I wrote 4 quarters.“ (4 rectangles ... 4 parts ... 4 quarters, each small rectangle (Fig. 3) represents an object named “the quarter“.)

Next, the teacher summarised the results of task 1 and 2. She asked: “Which of the

records 1, 4 and $\frac{4}{4}$ is correct?“ Even though the pupils suggested accepting all three possibilities, teacher A accepted only record “1“ for task 1 and “ $\frac{4}{4}$ “ for task 2. She explained her intention in the following discussion in the Czech team: “The pupils should understand that the whole is 1 and that 4 quarters make a whole.”

Teacher A gave pupils a question that allowed for several correct answers, she herself, however, accepted only one of them. She rejected a whole range of acceptable answers that could lead to further considerations and to deeper understanding. When discussing tasks with pupils, she tried to lead them towards the “correct” answer in different ways such as raising her voice, using a different intonation of the question, etc.

The teacher’s idea was to guide the pupils towards the independent discovery of the concept of fraction. During a common reflection of the video recording of performed lesson in the meeting of the whole team, it transpired that she did not take into account the experience the pupils might have with fractions, i.e. their pre-conceptions. It even appeared that she refuses the idea of pupils having pre-conceptions at all. Moreover, her knowledge of the topic (notion of fraction) was not good enough. Considering her insufficient mathematical knowledge, her initial intention could not be realised as she was not able neither to decide about the correctness of the pupils’ answers, nor to summarise results of discussions. Her uncertainty could be seen in her long commentaries that often included meaningless “padding”. It is documented by the following part of the protocol in which only the teacher is active. It is almost her monologue.

Teacher A: Try to write in a mathematical way what you did.

We can see this. *(She circles 1 in the record on the blackboard.)*

This question is still open. *(She points to the record $\frac{4}{4}$ on the blackboard. She points to number 1 on the blackboard again.)*

We have justified this. *(She goes to the magnetic board and points to the stripe divided into 4 parts.)*

Now what about this? Try to write it down. Think.

Andy *Andy wrote “4” on his small blackboard.*

Teacher A Andy, I do not want to go back to it, we have already crossed this out that it is not that. ... Even though I know what you mean. Try to think. What are we working with? With fractions. With some parts, you said, one whole. The same parts, you said. Try to think.

This *(She points to the stripe divided into 4 parts.)*, try to express this.

How much was this? *(She points to the non-divided stripe.)*

The video recording of the lesson² clearly shows that the uncertainty of the teacher and pupils was growing during the lesson and that the pupils were losing interest. The impression of the lesson was unsatisfactory both for the teacher and for the pupils.

Unit fractions and whole/part relation in teacher B's class

Teacher B decided to adapt the context (the stripe of paper) and used it (a) for the improvement of grasping whole/part relation, (b) for the work with unit fractions, and (c) for the development of idea of multiple representations. Here, we will outline the idea, tasks and pupils' reactions connected with points (a) and (b).

Description of the situation: Each pupil will get three congruent paper rectangles. One will be labelled $\frac{1}{2}$, one $\frac{1}{3}$ and the third $\frac{1}{4}$. Everyone will also get three different paper stripes so that it is easily seen that a small rectangle is one half of one of them, one third of the second and one quarter of the third (see Fig. 4).

The teacher's formulation of task: "You have three congruent paper rectangles. One is one-half, second one third and the third one quarter. How is it possible?" (She did not mention the big rectangles – stripes of paper.)

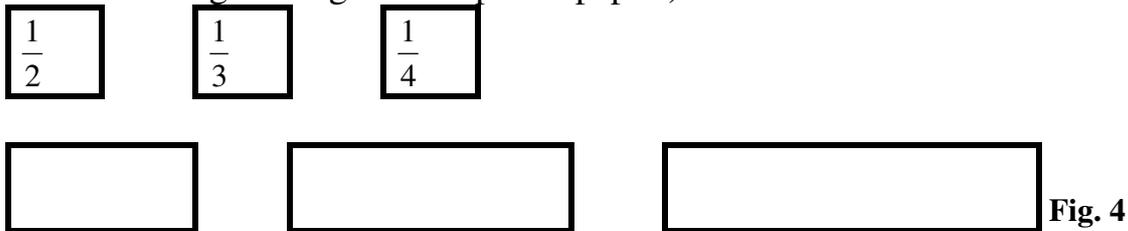


Fig. 4

Illustration from the dialogue (pupil – pupil(s) communication appeared only rarely):

- Teacher B How is it possible that one rectangle is one third, one is one half and one is one quarter if they are all the same?
 Try to think for a while. ... Hana?
- Hana Because each time it is from different paper and the paper has a different size. (*She bears in mind and shows, that they divided the paper stripes, rectangles, of different length into equal parts. Fig. 4.*)
- Teacher B And what about you, Tom?
- Tom Each of the parts is smaller but the big picture is the same.
- Teacher B Which picture?
- Tom The original shape, which is divided into the parts.
- Teacher B It is the same?
- Tom Ahem. Only that I divided it into smaller or bigger parts. (*Other pupils*

² We do not include transcripts of long parts of the lesson, as they would be unintelligible for the reader and require a detailed commentary, which is outside the scope of this text. We are only trying to characterise the course of the lesson.

raise their hands.)

Teacher B Children, does anyone want to say something about it? Jirka.

Jirka The original shape is always bigger and this one, for example, (*He takes the appropriate big rectangle.*) are three “one-thirds”.

Teacher B Jirka realised why you have the various long stripes of paper on your desks. Look at them. Is it somehow connected to the fractions?

Pupils Yes, yes.

Teacher B And how is it connected?

Marta As if the one third goes into the rectangle. (*She takes the appropriate stripe of paper.*) Into the smallest, the half and the quarter ... For example, the one third can go into this one three times.

Teacher B Do you want to add something?

Robert To assign them...

Teacher B Well, assign ... or what do you think?

Adam They are wholes for these. They are the wholes of the thirds ...

Teacher B ...of the quarters, of the halves ...ahem. Try to find the correct wholes.

From the discussion, it is obvious that teacher B knows where she is going and let the pupils explain what is unclear and overcome obstacles. Her good subject-didactic competence (in sense of Helus, see p. 2) enables her to react to the pupils' statements. If they give an answer, which she does not expect but which nevertheless brings new impulses, she is able to use it as a starting point for discussion and to adapt the planned procedure so that she still gets to the looked-for target. If the answer is not correct, she does not show her dissatisfaction. She shifts the initiative to the pupils. She tries to reach the conclusion and challenges the pupils to formulate it. We can illustrate it with the discussion devoted to “one third” and visual representation:

Teacher B Why did you choose it like this?

Vera Because if one third, then the whole ... there are 3 parts there.

Teacher B How many thirds can go there?

Vera Three. (*She waits.*)

Teacher B Go on.

Vera And that is one quarter, so it goes there four times.

Teacher B Now, we will demonstrate it. (*The pupils work.*)

Honza and Kacka demonstrated it, but in different ways. Which one is better?

(*Honza puts it ON.* Fig. 5) (*Kacka adds ALONGSIDE.* Fig. 6)



Fig. 5



Fig. 6

The teacher then asked the class to discuss the demonstrations suggested by the two pupils and to summarise their work.

It is necessary to pay attention to the understanding of whole/part relation and creating imagination of fraction for a long time (it was said many times). Here we recorded only small part of that period. In spite of this, we can observe a good level of subject-didactic competence of the teacher B there, which enables her to ask question without hesitation and react on unexpected input from the pupils.

DISCUSSION AND CONCLUDING REMARKS

The course of teaching experiments realised by teacher A and B confirmed our opinion that subject-didactic competence (in Helus's sense), is indispensable (if not decisive) for a good teaching of mathematics. The content of mathematical education at the primary school level is seemingly simple. However, what we deal with is the propaedeutic of different topics and thus, deep knowledge of concepts connected to them is necessary.

If we compare the lessons of both teachers, we can see big differences, which make apparent different levels of subject-didactic competence in various areas

- being aware of the goals of mathematics teaching,
- level of mathematical knowledge and ability anticipate pupils' answers, to react to pupils' reactions, evaluate their correctness and use their input,
- ability and need to lead pupils towards summaries of results and their justification,
- creation of creative and challenging climate.

The way the pupils from teacher A's class spoke and answered questions shows that their pieces of knowledge are random, discrete, disconnected. They practised a pseudo-skill – to be able to give an expected answer. On the other hand, the pupils in teacher B's class gave the impression that they were trying to integrate new knowledge to the knowledge acquired earlier, to make a network.

Already in the initial discussion of the whole team, it was apparent that the level of subject-didactic competence of teacher B in the chosen area (part/whole relation, fractions) is good and much deeper than that of teacher A. Teacher A, as we saw her, wanted to work on the development of her teaching, mainly to change her approaches. She said: "...to change my teaching from impressive teaching (using many different methods, teaching aids, topics ... in one lesson) to the effective teaching." The low level of her mathematical knowledge (at least of some topics) prevents her from reaching this goal.

In our opinion, if we want to apply principles of constructivist teaching, the importance of the level of subject-didactic teacher's competence is growing. It is

clear that the teacher who wants to be a creator of climate and bearer of challenges must have good knowledge of both content and its didactic elaboration. It is challenge for educators of both in-service and future teachers. Qualified pedagogical reflection can motivate changes not only in methodical procedures but also in approaches to instruction generally. But in our opinion it is necessary to nurture it and cultivate it in order to improve education.

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TEACHERS' IMPLEMENTATION OF A CURRICULUM REFORM

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Abstract: *This is an ethnographic study of four teachers' interpretation of the current mathematics curriculum in Norway, Reform 97, and their implementation of it. The relation between the two is essential in the study. Through focus group interviews, classroom observations, conversations with teachers and teachers' self estimation, I found different degrees of coherence between what teachers say they do and what they actually do in their classrooms. One key issue that emerged was that despite when a teacher's intention is to implement a reform; the way it turns out in the classroom becomes quite traditional. This raises issues of how teacher education needs to enable teachers to interpret visions into practice.*

Keywords: Classroom observation, Investigative approach, Constructivism, Teaching practice, Teachers' beliefs, Teacher-students' interactions, Ethnographic approach.

Objectives and significance of the study

Reform 97 is the educational reform in Norway that took place in 1997. As part of the more wide-ranging Reform 97, which affected the whole of the compulsory education system, a new curriculum was implemented in August 1997. New textbooks based on the curriculum were written. This curriculum or syllabus for grade 1-10 (age 6 to age 15) is referred to as L97 (Hagness, Veiteberg, Nasjonalt læremiddelsenter, & Kirke- utdannings- og forskningsdepartementet, 1999).

A research study (Alseth, Brekke, & Breiteig, 2003) suggests that the mathematics curriculum is not implemented as intended. Studies comparing pupils' performance on mathematical tasks before and after Reform 97 show that both in grade 7 and in grade 10 pupils perform lower in 2001 and 2002 than in 1995 and 1994 respectively (Alseth et al., 2003; Kleve, 2003). A classroom study revealed that teaching mathematics to a great extent still follows a very traditional pattern. Specified skills are in focus, and the entirety of the subject is rarely addressed relationally or holistically. Skills are drilled rather than understood. This is in great contrast to what is intended in L97 where students are supposed to develop their own mathematical concepts, and skills are supposed to be based on understanding and general concepts and principles within the subject.

Based on this, my research questions are:

- How are teachers in their mathematics teaching practice responding to the L97's recommendations?

- What kinds of interactions between the teacher and the student are observable in the mathematics classroom?
- How are teachers' practices in the classroom related to their beliefs about teaching and learning mathematics and to their goals for students in the subject?

Underlying theoretical framework

The curriculum describes different working methods in all subjects in general and in mathematics in particular. According to my interpretation of the curriculum, it encourages an investigative approach to teaching. It stresses that the pupils shall be active in the learning process. They shall be experimenting and exploring and through collaboration with each other acquire new knowledge and understanding. When reading L97 I find verbs like experiment, experience, explore, wonder and reflect. These initiate activities that I will say are investigative. For me investigative mathematics includes creativity, exploring activities and experimenting. It is essential that the students have an active role in the learning process. They should work on open ended tasks including problem solving. Justification and reflection are important. Students are working in groups rather than individually. It is not direct instruction from the board and not individual seatwork on exercising skills and procedures. I believe that an investigative approach to mathematics to a large extent develops conceptual knowledge. The following examples from other researchers' works on what an investigative approach implies support my findings:

In her study, Jaworski (1994) outlines common features of the mathematics classroom that to her seemed investigative; The tasks were inviting inquiry, and encouraging conjectures and justifications; The student's thinking process was emphasised; The class was organised mainly in groups; Many of the activities made use of physical objects; The teacher spent the time listening to and talking *with* the groups. Smith Senger (1998/1999) uses the term "reform curricula" with emphasis on problem solving and reasoning, use of manipulatives and technology, group work and communication. The role of the teacher is being a guide, listener and observer rather than a traditional authority and answer giver. Ernest (1998) compares investigative mathematics, or reform mathematics, with research mathematics. He argues that the introduction of investigational work in school mathematics involves a major shift in rhetorical style. "*For instead of representing only formal mathematical algorithms and procedures, with no trace of the authorial subject, the text produced by the student may also describe the judgements and thought processes of a mathematical subject*" (p. 257). Norton, McRobbie, & Cooper (2002) studied several teachers' responses to an investigative mathematics syllabus, "*Curriculum documents that are investigative reflect theories of learning consistent with major elements of social constructivist theory*" (p38).

When reading the mathematical part of the curriculum, L97, I see several principles which I interpret as reflecting a constructivist view on the teaching and learning of mathematics; Pupils shall be encouraged to build up knowledge largely by

themselves, they shall be active, enterprising and independent and they shall acquire new knowledge by exploring and experimenting. This seems to indicate an individual approach to learning reflecting Piaget's notions about assimilation and accommodation. L97 encourages discussions and reflections and emphasises how pupils' misconceptions and occasional mistakes provides ground for learning and insight in a constructive and confident atmosphere.

My reasons for interpreting L97's position as a constructivist one is underpinned by other researchers' writings about constructivism:

Confrey (1990) outlines several implications a constructivist theory has for teaching. The rejection of the assumption that one can simply pass on information to learners, the research on pupils' misconceptions and the use of misconceptions to promote in pupils to develop more powerful constructions and the fact that pupils themselves are supposed to construct their understanding through a process of reflection, are all essential components in her interpretation.

Noddings (1990; Noddings, Davis, & Maher, 1990) claims that "*cognitive premises of constructivism can dictate only guidelines for good teaching*" (p. 15).

Noddings et al. (1990) claim that learning mathematics requires construction and that constructive work in mathematics is necessary to learn it. Activities like hypothesise, try things out, execute mathematical procedures, defend results and reflections seem to be fruitful in such constructive work. According to Noddings et al there exists no orthodoxy in what a constructivist view on teaching and learning mathematics implies, but the emphasis on mathematical activity in a mathematical community, like in L97, is a common thread.

According to Jaworski (1994), constructivism is a philosophical position on knowledge and learning; it says nothing about teaching. Nevertheless she recognises that certain constructivists see implications for teaching. For example she cites 5 consequences from von Glasersfeld (Glasersfeld, 1985) that a constructivist perspective has for educational research: 1) He distinguishes between generating understanding and repetition of behaviour. 2) He focuses on what is going on in the child's head rather than overt responses. 3) He pinpoints that knowledge cannot be linguistically transferred, but that language can guide the child's construction of knowledge. 4) He explains how teachers should be more interested in children's errors (misconceptions). 5) He focuses on the importance for the teachers to be able to get inside the child's head not only by inferring the students' conceptual structures and operations but also in finding ways and means of modifying them. These are all principles reflected in L97.

To understand teachers' behaviour, Noddings et al. (1990) claim that teachers' beliefs about teaching and learning mathematics should be taken into account when conducting educational research within a constructivist framework. Because it is teachers' own beliefs about the nature of mathematics and about the teaching and learning of mathematics that determine whether opportunities for learning

mathematics is created in the classroom. Goos, Galbraith, & Renshaw (1999) place teachers and their beliefs about mathematics in a pivotal role. Teachers' beliefs influence the features of the classroom environment they create. However, they acknowledge that there exist constraints and pressure that may prevent teachers from acting according to their beliefs. This is in accordance with some of the findings of my research which is outlined later in this paper. Goos et al. (1999) identified three core beliefs from interviews with their teachers; Students learn mathematics by making sense of it for themselves; Teachers should model mathematical thinking and encourage students to make and evaluate conjectures; Communication between students should be encouraged so they can learn from each other. One of the core beliefs expressed by one of the teachers in my research was that "Students learn mathematics by finding out things themselves". In the analysis and finding section of this paper, I compare this belief with what I actually saw in that teacher's classroom.

Thompson (1992) distinguishes between knowledge and beliefs. She claims that beliefs can be held with varying degrees of conviction and often characterised by a lack of agreement over how they can be evaluated or judged.

Eisenhart, Shrum, Harding, & Cuthbert (1988) describe a belief system as a set of non contradictory beliefs which limit dissonance, contradiction and chaos and that individuals reluctantly give up their beliefs because that can cause cognitive disorder. They claim that if teachers shall change their practice, the desired change has to be related to teachers' beliefs about teaching and learning mathematics. Educational reform programs should take teachers' existing beliefs into account because educational reform programs "are unlikely to accomplish their goals unless they are first made compatible with or translatable into existing belief system" (p. 52). According to this, teachers' beliefs about teaching and learning mathematics will have implications for success or failure of educational reform programs.

According to Pehkonen & Törner, (2004) the process of how a belief is adopted is not well defined. Like Eisenhart et al. (1988) they assume that an individual's belief form a belief system. One such belief system is his or her views of mathematics, and they address four main components to views of mathematics which also are relevant for mathematics teaching; beliefs about mathematics, about oneself as a user of mathematics, about teaching and about learning of mathematics. Dionne (1984) used three perspectives of mathematics to identify belief systems: Traditional (Doing mathematics is doing calculations, using rules, procedures and formulas), Formalist (Doing mathematics is writing rigorous proofs and using precise and rigorous language) and Constructivist (Doing mathematics is developing thought processes, finding relations between different notions and building rules and formulas from experiences). Pehkonen and Törner label these perspectives as similar or corresponding to Ernest's (1991) three views on mathematics; Instrumentalist, Platonist and Problem solving. To investigate teachers' beliefs about teaching of mathematics, Pehkonen and Törner used Dionne's three perspectives in terms of Toolbox aspect T (Mathematics is a toolbox. Doing mathematics means working with

figures, applying rules, procedures and using formulas), System aspect, S (Mathematics is a formal, rigorous system. Doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts) and process aspect P (Mathematics is a constructive process. Doing mathematics means learning to think, deriving formulas, applying reality to Mathematics and working with concrete problems). They asked teachers to distribute a total of 30 points corresponding to their estimation of the three factors Toolbox, System and Process in which they estimated their own actual teaching and their value of ideal teaching. I used the same three aspects in my research. This is outlined in the section below.

Research design and procedure

In my study, I am focusing on teachers' interpretation of the L97 curriculum and on their implementation of it. The relation between the two is essential. Based on focus-groups and interviews I investigate how teachers' practices are related to their beliefs about teaching and learning mathematics and to their goals for their students. I contacted headmasters in 4 different schools and asked them to invite mathematics teachers to focus group meetings. Based on these meetings I selected 4 teachers, A, B, C and D, fitting some criteria: The teachers had to agree, I wanted both female and male teachers and also teachers with different educational backgrounds.

I am using research methods fitting largely into an ethnographic approach (Bryman, 2001). A simultaneously use of several data-gathering methods give me the opportunity to grasp a complex reality (Pehkonen & Törner, 2004). I have used focus groups interviews to get information about what teachers say about L97 and how they relate their teaching to what is said in the curriculum. In analysing the data from my focus groups it is important for me to be aware of the different levels of information the data give. On one level teachers speak from their inner thoughts and meanings, struggling to express what are really inside their head. On another level they speak from what they know as a teacher and what they say is deeply embedded in social practices of being a teacher, it is very socio-culturally rooted. A third level can be rhetoric; The teacher knows who I am, and tries either to express what he thinks I want to hear or since he knows what the curriculum says, he expresses that or he challenges that. In these cases the teachers respond to me and who I am rather than to whom they are. I tried to make the teachers speak from their experience. Krueger & Casey (2000) encourage use of questions leading persons to speak from experience; by *[asking] participants to think back* (p.58) rather than wishes for or what might be done in the future. That increases the reliability since it focuses on the past.

I have observed four chosen teachers one lesson a week for 3 months, and I have had conversations with them before and/or after the lessons. All of this has been audio taped, and parts of it are fully transcribed. From other parts I have written summaries. I have also written field notes; both from students' activities, teacher's presentation and teacher-student interactions. These notes are useful in addition to the transcripts or summaries because they give information about what can have happened that not

necessarily has been recorded. Most of the time, I wrote down my immediate reflections after a lesson. The purpose of the conversations with the teacher after a lesson was to clarify what had happened within the lesson. I also asked the four teachers to write down, about one page long, their personal opinion of what good mathematics teaching is.

I have also information from the teachers obtained through questionnaires and self estimation. In the self estimation form I drew on the same three aspects as Pehkonen & Törner (2004). My four teachers were asked to distribute 30 points corresponding to their estimation of the factors T (toolbox), S (system) and P (process) in which they should value their actual teaching, what they think is ideal teaching and in also what they think L97 reflects with regard to these aspects.

Analysis and findings

In this paper I will limit my analysis and findings to one teacher, Cecilie. She has been a teacher for 8 years, and this year she teaches two 10th year classes (16 year old students). I attended one of the classes once a week. Cecilie (C) says open-heartedly that she likes L97:

C: When I start a new topic, I read L97 and I have made notes from books I have read that I can use

BK: How did you react when L97 first came?

C: I liked it, but I didn't like the textbooks following it. I think they fitted the old curriculum.

BK: What do you especially like in it?

C: That it focuses on methods and deriving formulas. The way I see it, students are supposed to explore things themselves, using play for example. It becomes more exciting that way and I believe they learn some mathematics they won't learn by learning formulas by heart.

Cecilie estimated her own teaching with 10, 8 and 12 points to the tool box-, system- and process- aspect respectively. Her view of ideal teaching was 5-10-15 respectively to the same aspects. Ten points to each aspect is how she sees the curriculum. The table below presents an overview of Cecilie's estimations.

Cecilie	Mathematics as a toolbox	Mathematics as a system	Mathematics as a process
My real teaching	10	8	12
Ideal teaching	5	10	15
L97's view on teaching mathematics	10	10	10

So far these findings tell us that there is a coherence between what Cecilie says she believes in and what she intends to do in the classroom. On this macro level which

addresses my first research question, I find a teacher whose intentions are to implement L97 in an investigative way. I also found that she chose untraditional subject content as exploring Pythagorean triples, working with tokens and figure numbers, mathematical aspects of the calendar and proofs. She often linked the subject content to mathematics history, which is encouraged in L97.

Cecilie teaches both from the board (plenary lessons) and individual students or pairs of students while they work on their own (seat work). I never saw her organising group work. However, sometimes more than two students worked together.

In analysing data from classroom observations I am drawing on a study by Mortimer and Scott (Mortimer & Scott, 2003) in science education. They identified two main communicative approaches; dialogic and authoritative, and they saw an interactive or a non interactive process within each approach. This leads to the following analytical framework in my analysis:

- Authoritative – Interactive approach: Teacher presents mathematics by leading students through easy manageable questions.
- Authoritative – Non interactive approach: Teacher presents mathematics without any interactions with the students.
- Dialogic – Interactive approach: Teacher poses genuine questions to encourage constructions from students' ideas.
- Dialogic – Non interactive approach: Teacher presents mathematics from different points of view.

The following episode from one of Cecilie's whole class introduction illustrates the first one of these communication approaches:

The teacher starts by drawing a right angled triangle on the board with the measure of two of the sides (3.6 and 4.8). (T is Cecilie, S and other letters are students)

T: I hope you've got your calculators. You'll need them doing this task

T: What kind of triangle do we have there, M

M: Right Angled triangle.

T: Then we know the lengths of two sides. And now I don't use unit. We are only interested in the numbers. How can I find the third side, L?

L: You have to use Pythagoras.

T: Yes. Have to use Pythagoras. Let us try to do that with this triangle. If we call this side for x, L?

L: Must take $x^2 = 3.6^2 + 4.8^2$ (T writes on the board)

T: Yes, let's calculate that. Three point six squared is?

S: Twelve point ninety six

T: Four point eight squared is?

S: *Twenty three point o four*

T: *(repeats 23.04 and writes it on the board) The sum of these numbers is?*

S: *Thirty six*

T: *It is thirty six (writes on the board).*

S: *It makes six*

T: *Yes. Okey. It became six long. This was lots of calculations. If we look at the numbers here we might have simplified it. Is it like, here I have added one point two, and if I add another one point two I'll get the third side? Is that a rule always working? Let us take another example. New triangle (She draws a new triangle on the board with sides like 7.5 and 10). If that is seven point five and that is ten, will that one be twelve point five? Can you check if it works?*

BH: *Yes*

T: *That worked as well. Your exploratory task is now: Does it always work? Does it work for any length? Find new lengths on the smaller sides of a triangle and check if it works on any side.*

S: *But I didn't understand what you did?*

I have underlined the questions the teacher poses to the students to pinpoint the kinds of questions she asks. I find that relevant with regard to the communicational approach in this episode which I interpret being authoritative. The interactions consist mainly of easy manageable questions to control students' attention or to review subject content. The teacher is not encouraging constructions from students' ideas. She is not offering a possibility for students' ideas to come up. The teacher's presentation is in a dialogic format as she invites students to participate. However, the nature of the questions she asks demonstrates the communication being authoritative. There are interactions between teacher and students, but the interactions are within the teacher's system of understanding. One student says: I didn't understand what **you did**. The students are only answering easy manageable closed questions and the "exploratory" task she gives them boils down to find out what the relation between the two shorter sides in a right angled triangle has to be if the teacher's claim shall work.

When comparing the micro analysis of this teacher's lesson with a macro level account of this teacher, there is a tension, a gap. She likes the curriculum, she believes in an investigative approach. According to the estimation form she looks upon mathematics to a great extent as being a constructive process. Her intention is for the students to explore mathematics and her choice of topic is non- traditional. However, the way it turns out in the classroom with the students is authoritative and traditional. She is working within her system of understanding mathematics and not encouraging constructions from students' ideas.

This indicates that implementing a curriculum reform is not straight forward and very hard despite that the teacher's intention doing it is present and it raises issues of how teacher education can enable teachers to interpret visions into practice. Williams &

Baxter (1996) report similar findings with regard to discourse-oriented teaching. Even in a class-room with a teacher who had made progress toward a reform, the classroom returned quickly to a more traditional orientation. Prawat (1992) also reports that desired changes in classroom practice into a reform-oriented one failed to appear although the teacher showed willingness to experiment with the innovations called for.

Inspired by the discussion at the Cerme 4 conference, I will conclude this paper by presenting some comments to my findings. It is probably not sufficient to explain the gap between what teachers say they do and what is observed by an observer in their classroom by just saying that we have to promote a change in teachers' beliefs. One reason for the tension found may be that teachers can only teach in ways they know how to teach. That is often in ways they have been taught themselves. Another issue discussed was that despite that an observer finds that there is inconsistency between a teacher's intentions and what is observed in the classroom, for the teacher there is consistence (Skott, 2001). Skott claims that inconsistency in an observer's perspective does not do justice to the complexity in the classroom the teacher has to deal with, and that teachers cannot be inconsistent.

This tells us that there is not a linear cause-effect relation between teachers' beliefs and their practice. According to Thompson (1992) they are rather dialectically related and the extent to which a teacher's conceptions are consistent with practice depends on the teacher's capacity to reflect upon his/her actions. It is therefore of greatest importance to encourage such reflection in teacher education.

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TEACHERS USING TECHNOLOGY: MODELS OF THE COMPLEXITY OF PRACTICES

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Abstract: *Classroom use of technology reinforces the complexity of teacher practices by introducing a number of new factors. Our aim is to understand the impact of these factors on systems of teachers' practices, and the conditions for classroom use of technology. We consider Robert and Rogalski's (2002) "dual approach" and we try to complement this approach by using models dedicated to teacher use of technology: Ruthven and Hennessy's (2002) model addresses teachers' views of successful use, whereas Monaghan (2004) develops a model of teacher classroom activity. In this paper we test these models as means to analyse actual teacher practices in two fields: teachers at lower secondary level using dynamic geometry and teachers at upper secondary level non-scientific stream using a spreadsheet.*

Keywords: teachers, models of teacher activity, technology, spreadsheet, dynamic geometry.

Introduction and aims

A growing number of research studies in Mathematics Education focuses on teachers' practices and classroom activity, and on their influence on students' understanding and learning of mathematics. The complexity of these practices and influence is generally recognized.

In the area of the use of technology to teach and learn mathematics, a similar movement is observed. Research studies in this area have existed for more than twenty years. Many technology-based innovations for improving teaching/learning have been imagined and tested by research methodologies. A common impression is however that there is a wide gap between these innovations and the actual situation of classroom use of technology. The small number of teachers really using technology (Jones and Lagrange 2003) is a concern because it seems out of proportion with the money authorities have spent to develop technology in schools. An additional concern is the way teachers use technology because it seems that they tend to try solving problems linked to their actual practices, rather than changing them in order to take advantage of the potentialities it offers (Ruthven and Hennessy 2002). Teachers trying to change their practices have encountered strong difficulties (Monaghan 2004).

Recently, Robert and Rogalski (2002) proposed a twofold approach to teachers' practices. The first component (didactical) takes into account the links between students' activities and the teacher's management. The second component (ergonomic) considers the teacher as a professional who is performing a specific job. Articulating these two approaches made it possible to see teachers' practices as a complex and coherent system, resulting from a combination of individuals' personal and professional history, knowledge and beliefs about mathematics and teaching.

As for us, we see the use of technology as the introduction of a number of new factors into these systems: re-evaluation of epistemological and semiotic aspects of mathematics (epistemology and semiotics), new relationship between learner and knowledge (cognition), evolution and diversity of the relationship between users and tools (instrumentation), impact on institutional balance (curricula, tasks, techniques...) (Lagrange et al. 2003).

Our aim is to study the impact of these new factors on systems of practices. This study is a necessity, as schools have to cope with social demands for integration into the information age. It is also for us an opportunity to understand better systems of practices by looking at perturbations that technology brings about and at possible re-equilibrations.

There are currently two strands of research related to teacher use of technology. One is about teacher training. According to Abboud Blanchard (1994), the nature of teacher education in technology constitutes a major cause of poor integration into school: offering teachers examples of innovation is not enough; teachers need didactical knowledge about the use of technology. In the same strand, research studies like Assude and Grugeon (2003) are centred on strategies and design of training methods, for a successful integration and the evolution of teaching. Teacher education is seen as a long-term process.

Another strand focuses on the analysis of teachers' practices. The purpose is to identify actual practices involving technology and to understand how they work. In this strand, Ruthven and Hennessy (ibid) took into account teachers' view of successful use of technology in order to elaborate models of their practices. Monaghan (2004) offers a model of analysis borrowed from Saxe (1991) to examine the influences of key factors on the activity of the teachers and to understand 'holistically' the complexities of practices.

Our work is presently in the second strand. In our view, studies of teacher education can produce evidence of the limitations of current strategies in this field. But, to build better strategies, more understanding of the conditions in which technology can exist in real classrooms is required. Thus, bringing new directions for teacher education is a long-term goal.

At a general level, we found especially useful Robert's and Rogalski's 'twofold approach', described above. We are interested in complementing this approach with models intended to study teacher practices involving technology. Models are

important to understand a complex reality, and testing models in relationship with experimental data can help to progress towards this understanding. We choose Ruthven-Hennessy's and Monaghan's models because they were developed to study 'ordinary' classroom use of technology and because they seemed consistent with Robert's and Rogalski's approach to teacher's practices.

Although classroom use is not very developed, there is a lot of different ways technology can be integrated. We had to choose fields where enough 'real life' teachers use technology. We found two fields: one is dynamic geometry at lower secondary level and the other is spreadsheet at upper secondary non-scientific level.

In these two fields, teachers are under a strong institutional pressure: dynamic geometry is much encouraged at lower secondary level and spreadsheet use is compulsory at upper secondary level non-scientific stream. Thus, the two fields offer good opportunities to test the models against actual teacher practices. In other fields, for instance the use of Computer Algebra, there is a huge research and innovative literature but few real 'ordinary' classroom implementations.

Data and Models

Our investigations are being carried out as parts of doctoral studies. This section explains in more details the models of teacher activity that these studies consider and presents the empirical data.

Models

Ruthven and Hennessy (ibid) offered "a practitioner model of the use of (technology) to support mathematics teaching and learning" built from a statistical analysis of themes occurring in interviews with teachers. A first set of themes (engagement intensified, activity effected, ideas established) "corresponds closely to ultimate teaching aspirations – of participation, pace and productivity, and progression". Other themes stress more directly the potentialities of technology: ambience enhanced, routine facilitated, features accentuated. The two sets of themes are not directly connected. The connection is made through 'intermediate' themes related to processes that, according to teachers, realise the potentialities of technology: motivation improved, restraints alleviated, attention raised.

This model helps us to understand the consistency between teachers' view of the use of technology and pedagogical concerns, contrasting with the emphasis generally put on changes that technology should bring into epistemology and learning processes. Pedagogical constraints in classroom activity are very influential in forming this view and teachers emphasize features that can turn technological potentialities into means for a more stable and effective classroom activity.

Although a direct influence of established practices is apparent in this model, it is also compatible with teachers' awareness of the potentialities of technology impacting on these practices. This is especially the case when teachers mention a potentiality for "student tinkering" that the statistical analysis shows not connected

with “teaching aspirations”. As the authors say: “*As well as serving as a ‘lever’ through which teachers seek to make established practice more effective, technology appears also to act as a ‘fulcrum’ for some degree of reorientation of practice, and a measured development of teachers’ pedagogical thinking.*” (ibid p.85).

Monaghan’s work is more centred on teacher classroom activity and he proposed an adaptation of Saxe’s (1991) model to study this activity. This model was introduced to explain how an individual’s knowledge is shaped and organised by experience as well as structured by logical systems. It aims also to understand how artefacts and forms of social organisation ‘are intrinsically related to the nature of (individual)’s understanding’ (ibid p. 5). Saxe’s study of the individual’s activity in relationship to knowledge development starts from an analysis of ‘emergent goals’: “*Goals are emergent phenomena, shifting and taking new forms as individuals use their knowledge and skills alone and in interaction with others to organize their immediate contexts.*” (ibid pp. 16, 17).

Four parameters are likely to influence these goals and Monaghan provides interpretations of these specific to the situation of a teacher:

Activity structures: the general organisation of a course and of the teacher’s and students’ tasks.

Conventions – artefacts: with or without technology, mathematics teaching involves cultural artefacts like algebra for instance.

Social interactions: relations between teacher and students.

Prior understandings: teachers’ mathematical, pedagogical and institutional knowledge.

Unlike Ruthven and Hennessy’s, Monaghan’s study is based on observation of classroom practices. This observation helped him to study the impact of the use of technology on each of the parameters. Monaghan showed how this use greatly changes the overall balance of teacher classroom practices and the interweaving of parameters: they evolve together and a single one cannot account for emergent goals.

Experimental data

Our methodology of observation addresses both components of the twofold approach. We try to characterize the didactical component by reconstructing the organisation of student tasks that a teacher more or less explicitly planned and by analysing the way he(he) carried out this organisation during his(her) classroom activity in relationship with the opportunities this organisation brings for students’ learning. We look for the ergonomic component by collecting data on the teacher’s representations and previous experience both in the use of technology and in mathematics teaching. This methodology implies that our studies are mainly clinical.

During the investigations related to the use of dynamic geometry, teachers expressed rationales for using technology consistent with Ruthven and Hennessy’s model while

we observed discrepancy between these rationales and the task offered to students as well as the actual classroom situation. 17 lessons were observed for 4 teachers. We choose here to analyse a lesson where this discrepancy was striking. We have called the teacher Anne.

At upper secondary level 15 lessons were observed for 3 teachers. We had the opportunity of comparing two teachers teaching the same course. In our report we have called one Beatrice and the other Charlotte. This comparison was an opportunity to put into operation Saxe's model as adapted by Monaghan, with the aim of seeing how it could help to discriminate between the two teachers' profiles.

Anne's view and practice of dynamic geometry

Anne had been a teacher for 12 years and had used technology with her students on a regular basis for 4 years. In contrast to other teachers whom we observed using a single computer and a video projector, her use of technology privileged students' work in a computer room. She was trained in the use of Geoplan a dynamic geometry system. Geoplan is a French development, with features not entirely alike Cabri's. Anne used also a booklet of Cabri activities.

Students' task and Anne's view of the contribution of dynamic geometry

We analysed the worksheet that Anne prepared for her 7th grade class. The objective was to introduce students to the topic of the circumcircle of a triangle and more precisely that they understand the position of the circumcenter. The worksheet offered six steps. The first four steps were a guided exploration and the final two asked for a theorization: writing out a property and designing a program for constructing a circumcenter. At each step, students had to do constructions with Geoplan, to move objects, observing and writing out answers. This activity took advantage of the 'dragging' feature, making dynamic geometry a support for exploration and theorisation. The worksheet did not consist of 'push this button' instructions. Students were introduced to Geoplan in a former session and were supposed to have sufficient knowledge of the software.

The analysis of Anne's interview showed that she saw the students' activity mainly as a 'transposition' from paper/pencil to the computer screen. According to her, the affordances of the software are primarily the speed and the accuracy of drawings. She thought that with this tool the students were not likely to make mistakes when drawing as they would do in paper – pencil, for instance confusing perpendicular bisector ('médiatrice' in French) and median, because they only had to transfer the instructions from the sheet to Geoplan. Anne thought that students could use Geoplan 'easily'. According to her, they should not get lost in the menu items because of its clear organisation. When asked for possible difficulties, Anne just mentioned that some students might have forgotten what they learnt before about Geoplan.

Anne's pedagogical organisation of the session was quite elaborate: two half-classes worked concurrently in the same classroom, one on the computer task described

above and the other on textbook paper/pencil exercises. The two half classes changed role half way through this 45-min session. Anne cleverly planned this organisation to deal with the limited number of computers. It is however important to notice that she had to restrict her activity mostly to interactions with a single student. But, according to her, it was not a problem because her students could learn quickly and although they had to work mainly alone because of this special organisation, they were able to achieve the task.

Observation and discussion

During the session, we observed Anne's individual interactions with students working on a computer. There were 64 in 45 mn, confirming that they constituted the majority of her activity. We classified them into four categories: "creating an object using Geoplan (75%)", "making sense of an unexpected computer outcome (5%)", "understanding the worksheet (6%)", "writing out answers (14%)".

Three interactions in four were devoted to the first category. Even when students could access the menu items as Anne had expected, they had great difficulty in grasping the logic of the creation of objects in the dialog boxes of Geoplan. Anne's mediation was generally not directed towards understanding this logic. She rather tried to favour a quick operation in order that students could reach the 'mathematical' part of the activity and she provided technical assistance rather than a reflection about Geoplan's operation. Time was very limited and, in Anne's view, this part of the task was a pre-requisite to the observation of the properties and the deductions, but it was not really significant. Mediation was not always efficient because Anne sometimes confused features of Geoplan with Cabri's.

Our analysis brings into light Anne's great care over the pedagogical conditions needed to ensure a satisfactory classroom activity. The pedagogical design (two half classes working concurrently) showed great professional skill. In the rationales that she gave for the use of Geoplan we see clearly the way a teacher connects aspirations for improved classroom atmosphere and activity, with potentialities of technology through intermediate themes, as in Ruthven and Hennessy's model. In the actual classroom situation, the connection did not really work because of an underestimation of the need for students' understanding of Geoplan's operation. We see Anne's individual technical assistance to students as a way to re-establish the connection by 'scaffolding' students' use of Geoplan.

Another discrepancy between Anne's view and the situation is that the task offered to students involved potentialities of Geoplan that she did not mention: experimenting with concentric circles passing through the vertices of a triangle by moving the common centre to induce positions for superposed circles, is not just using a machine to get an accurate drawing. Anne told us that she designed the activity herself: our hypothesis is that she was influenced by materials about the use of dynamic geometry without being entirely aware of the underlying ideas. While the worksheet asked students to experiment with different positions of the common centre, Anne's view

seemed to be that a single accurate drawing would 'show' the position of the circumcenter.

We said above that we study the didactical component of a teachers' activity by considering the organisation of student tasks and the opportunities for students' learning. Here, although the worksheet seems to offer such an opportunity through a reflection on the critical positions of the common centre, student and teacher activity focuses on obtaining a drawing rather than on this reflection. When the expected contribution of technology to the classroom activity failed, the teacher tried to overcome the obstacle by becoming a mere 'technical assistant'. The constraints of classroom activity seem too strong to allow a reconsideration of practices in relation to learning opportunities offered by the technology.

Using Saxe's model to contrast Beatrice and Charlotte's practices

Beatrice (introduced above) was likely to have made such reconsideration, because of her extensive and long experience in the classroom use of technology. Charlotte, although an experienced teacher, was much less committed in the use of technology. Below, we compare the two teachers in terms of each of Saxe's parameters before analysing a lesson.

- *Activity Structures*: The course was two hours per week, one as a whole class and one as a half class. Beatrice taught the whole class hour in a computer room: students worked in teams with a computer at their disposal. She devoted the half class hour to a report of the teamwork and to a synthesis. Teams reporting their work could use a computer hooked to a video projector and to the local network. Charlotte taught the whole class in an ordinary classroom without the spreadsheet and each half class in a computer room. In both cases, Charlotte's students worked more or less individually following a worksheet.

- *Conventions - artefacts*: The use of a spreadsheet is compulsory in this course. In Beatrice's lessons, the students had a spreadsheet always at their disposal. However Beatrice's worksheets gave no instruction to use the spreadsheet or paper - pencil. The students decided for themselves. For Charlotte's students, when in the computer room, it was clear that they had to work on a spreadsheet. Moreover, Charlotte's worksheets were really specific about this use, referring to cells and formulas.

- *Social Interactions*: Beatrice's classroom management was not the same in the whole and half classes. In the whole class, when students were working in teams, Beatrice spoke infrequently and generally to encourage students to work as a team. In the half class, during the report of teamwork, she spoke much more, questioning the group and asking the rest of the class for their reaction. In contrast, Charlotte's interactions with students were similar in the computer and in the ordinary room. These interactions were very frequent (98 in a 50 mn session) and generally between herself and a single student. Students generally asked Charlotte to check their answer before passing to another question.

- *Prior Understandings*: Changing students' image of mathematics was Beatrice's first goal when using technology in this class. She explained that a majority of her students failed in mathematics, and thus her priority was to make a different entry in mathematics and to change the working atmosphere. She considered technology as a very strong means to motivate students for mathematics lessons. In Charlotte's view, technology was introduced in this course in order that students should learn about spreadsheets. "It is not for mathematics but for a general training" she explained to us.

Beatrice's lesson

We analyse here a fragment of an observation in a half class session. Beatrice's general objective in this session was to define two types of growth (linear growth and exponential growth) starting from the reports of teamwork on a series of tasks. This session was made up of four steps: the report of the work of a team about the first task (step 1), an introduction by the teacher to the concept of linear growth (step 2), the report of the work of the second team on another exercise (step 3), a short synthesis about the link between the multiplying coefficient and the exponential growth (step 4).

We consider the first step of the session and the 'emergent goals' (in the sense of Saxe) appearing. The task was to calculate the terms of a linear sequence in a situation. The sequence was defined in words (not mathematically) and explicitly (not recursively). The initial value was 100, and the common difference was 2. Students had to model the situation and, if choosing to use the spreadsheet, to produce a two column sheet like that in figure 1, where the formula was entered into cell B2 and automatically copied below.

Figure 1

	A	B
1	0	100
2	1	=100+2*A2
3	2	=100+2*A3
4	3	=100+2*A4

Figure 2

	A	B
1	0	100
2	1	=\$B\$1+2*A2
3	2	=\$B\$1+2*A3
4	3	=\$B\$1+2*A4

Beatrice's objective was to go farther and to consider a more general situation where the initial value could be changed easily. In a spreadsheet, using an absolute reference to a cell containing this value does this. Absolute references are written with '\$'. See figure 2.

Beatrice asked a team of students to present their work to the class and told them that they were free to choose an environment (spreadsheet or blackboard) for the presentation as in the teamwork.

The group started to present their work to the class, giving their interpretation of what the exercise said and calculating the first three terms of the sequence. At this time, Beatrice became aware of a difficulty: these students did not use a spreadsheet in their teamwork and were not keen to use it. Actually, all students in the class made

very little use of the spreadsheet in their team work, mostly trying to understand the exercises and doing some ‘by-hand’ numerical (not algebraic) calculation

We analyse this difficulty as an effect of contradicting parameters. Beatrice’s view of the role of artefacts and of social interactions was consistent: students, as individuals should be free to choose what they think is the appropriate tool and her relation to students should maintain this freedom. But this view contradicted her prior understanding of the role of technology in the relationship between students and mathematics. Her goal was to bring students towards some algebraic understanding by way of spreadsheet formulas. It is an ambitious goal but important for her.

Beside this goal, an emergent goal appeared: to make the students use the spreadsheet. The remarkable thing is the way she achieved this goal, by way of a dialog with the class, insisting on the “modern” aspect of technology.

Beatrice: What there now? They do all calculations by hand? There is more modern means to do that? There is a more modern means to do that, you make by hand?

Student: technology tool

Beatrice: that is?

Student: the spreadsheet

Beatrice: the spreadsheet, then go ahead.

Another difficulty appeared because students did not understand the need for the use of an absolute reference. They found that, to change the initial value, it was easy to change the first formula and copy it again. Again Beatrice had to find a way to motivate the students to use the spreadsheet in accordance with her goal. She introduced a new constraint: when the initial value is changed, one must change just one cell in the sheet.

In this fragment (20 mn) of the session, we see Beatrice confronted with two unexpected sub-goals resulting from conflicting aspects in her personal parameters. Charlotte’s parameters were simpler and more consistent. There was no question of students choosing to use or not use a spreadsheet nor any emphasis on a contribution to learning mathematics. Eventually she also encountered a difficulty when students had to enter absolute references. In contrast to Beatrice’s class, it was not something to be discussed. The worksheet prescribed the use of a special key to transform a relative reference but some students tried to do this transformation simply by keying ‘\$’ inside the formula. Charlotte did not know that the two operations are equivalent and she forbade students to key a ‘\$’.

Conclusion

Models do not miraculously open ways to successful integration of technology. But, combined with classroom observations, they can help to make sense of phenomena. For instance, it is a general observation that teachers teaching in a computer room devote much time to technical scaffolding, when they expected that technology

would help their students to work alone and that they could act as a catalyst for mathematical thinking (Monaghan, 2004). In Anne's case, Ruthven and Hennessy's model helps us to understand how a teacher can connect potentialities of a technology to her pedagogical needs, overlooking mathematically meaningful capabilities. The observation shows what happens when the connection does not work: the teacher tries to re-establish the connection by becoming a technical assistant.

It is also well known that the more complex and ambitious goals a teacher has the more his (her) classroom management will be difficult. Monaghan's model helps us to appreciate specific teachers' positions. Technological tools are flexible and their relationship with mathematics is subtle. In this sense Charlotte's position is unsatisfying: technology in the mathematics classroom cannot be just learning to use a spreadsheet in a closed way. Charlotte is nevertheless an experienced and conscientious teacher. She did not choose to teach this course and she tries to manage it as best she can, adapting her goals to what she regards as a tolerable complexity. In contrast, Beatrice's parameters reflected her high ambitions for this course and it is not a surprise that they sometimes conflicted. She had to make real efforts to get herself out of such conflicts.

Models do not either open direct route for teacher education. However, accounting for the complexity of practices with technology, they suggest that teacher training based on the transmission of 'good practices' will not work. Teacher Education has to consider the connection that teachers make between technology and their pedagogical preoccupation as in Ruthven and Hennessy's model while introducing reorientations of practices that could be compatible with a sustainable increase in the complexity.

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THE FRAMEWORKS OF SCHOOL EXPERIENCE: AN ANTHROPO-DIDACTIC APPROACH TO THE PHENOMENA OF MATHEMATICS TEACHING

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Abstract: *This research is based on the ethnographic observation of three school classes of children between 8 and 10 years old with the aim of studying the place and the function of ritual practices in mathematics lessons. It fits into a double framework – anthropological (customs, rituals), and didactic (memory and contract). The author shows how certain ritualising forms contribute to defining teaching situations and to structuring the action of pupils. Finally, he poses questions about the constraints imposed by didactic culture and the limits of over-ritualising teaching situations.*

Keywords: didactic of mathematics, anthropology, didactic contract, ritual, custom.

Introduction

The research under consideration here is part of an anthropo-didactic approach to the phenomena of mathematics teaching in elementary schools, in which the school situations are viewed as the product of a double structuring process: didactic structuring around the transmission of knowledge and the social “obligation” that the protagonists face (teaching for the teacher, learning for the pupil); and anthropological structuring around the non-didactic conditions in which this knowledge is transmitted (form of school, family culture, status and role of those concerned, pedagogical convictions, value system, etc.). These two structuring modes actively contribute to the definition of the teaching situation. By simultaneously taking account of the didactic conditions (objective, formalisable, necessary) structuring the milieu *and* of the anthropological conditions that are *a priori* non-didactic (the ways in which individuals are subjected to cultural forms), we can pinpoint the didactic effects that could not be perceived in just one of these frameworks.

Any teaching relationship is fundamentally dissymmetric with regard to the knowledge that is mobilised. The teacher is the one who occupies the position of transmitting knowledge that the pupil is *de facto* ignorant of: it is this ignorance that is the basis for and which justifies all teaching relationships. The didactic contract gives us an understanding of teacher-pupil games: the teacher cannot tell the pupils what he expects of them without stating what he actually wants (in other words, to use what he has taught them in a new situation); as for the pupils, they can only learn if they accept not to be “taught” (in the traditional sense of the term). This is what

made Brousseau (1998) say that learning is linked not to the drawing-up of a good contract but to breaches of that contract¹.

The teacher also needs the pupils to adhere to his teaching project. However, the pupils cannot adhere to it on the sole basis of the knowledge under consideration because for the pupils, that knowledge is pure fiction (Chevallard 1991). The teacher is thus forced to develop a number of strategies (staging, rituals, movements, attitudes, mimes, etc.) which will incite the pupils to commit themselves to an activity they do not control. We think that these approaches or behaviours, *apparently* routine and devoid of sense on the didactic level, need to be considered other than as residual categories, and that they contribute to making pupil commitment possible in teaching situations.

The aim of this research is to show how certain rituals actively participate in the set-up and progression of teaching situations. However, distinct variations in context can lead to modifications in ritual procedures, which can no longer play the same symbolic and practical role. A study of the effects of these disturbances on the “ceremonial” progression of lessons may be one of the ways of updating these incorporated, unconscious, yet very significant didactic forms which regulate school life. To analyse these phenomena, we will call upon the theory of didactic situations, along with the concepts of didactic memory and didactic contract (Brousseau, 1998) and the theory of frameworks (Goffman, 1991).

Methodology and area of research

The research involved the ethnographic observation of three primary school classes (pupils from 8-10 years) over four weeks for each class, in the mornings. It focused in particular on mathematics lessons. The observation took place in three phases: one in the presence of the class’s teacher; one in the presence of the teacher’s replacement (the teacher was on a training course); and one with the teacher when she got back from her course:

- The first observation phase (two weeks) enabled us to identify a number of ritual practices and to determine certain aspects of the “didactic culture” of the class;
- The second phase (one week) consisted in identifying and describing the rupture effects introduced by the replacement (modification of rituals, methods of adaptation and strategies of teacher and pupils);
- In the third phase (one week) we observed the procedures for the restoration of the previous didactic forms and/or the introduction of new teaching practices.

This approach offers the advantage of allowing us to observe the effect of the ruptures introduced by the change of teacher and the modification of ritual

¹ On the didactic contract, see the report by Sarrazy (1995).

procedures in "quasi-natural" situations. Here we will only deal with a few aspects of ritual practices noted in mathematics lessons.

Introduction to the lesson: change of framework

The introduction is a crucial phase which very often determines the conditions for the commitment of the pupils to the didactic situation. Here we are placed upstream of the devolution phase (Brousseau, 1997),² in other words before the lesson itself has begun or the instructions are formulated. The pupils are introduced to the new teaching situation, either by changing the subject being taught (switching from a reading activity to a mathematics lesson, for example) or by opening a didactic situation after a non-didactic phase or an interruption (recreation, for example). The teachers most often use two types of markers in changing the framework: with the ritual movements, words and objects only indicating a change of activity; ritual activities the content of which is directly linked to the nature of the teaching activity to come.

Ritual markers: words, movements, objects

When it is not introduced by a formal rite, the introduction of the lesson is characterised by simple ritual markers (nursery rhymes, chanting games or verbal signals: music, tambourines, bells), often used in infant school classes. The markers are often less visible in primary school classes and the signs that mark a change of framework are sometimes tenuous. They may be simple gestures (clapping of hands), ways of saying things, or ritual postures by the teacher, all of which suffice to create the conditions for entering into the new didactic situation.

Class 3: The teacher sits on a stool in front of the pupils and her attitude shows a certain solemnity (arms crossed, looking around the room); mobilisation of attention and imposition of silence: "*Right, quiet now everyone*". The intonation of her voice (firm) is sufficient to indicate a change of framework; the pupils modify their posture, thus signifying that they are adhering to the new situation. Instructions are given quickly and the activities begin immediately.

The ritual plays an instrumental, prescriptive and normative role which creates, with maximum economy, the conditions for teaching. It does not only impose silence and a respectful attitude on the pupil, it also marks and defines a new situation (a new framework) in which everyone will then take their place in their own specific way. We can consider that by marking the boundary between "before" and "after", these ritual activities play the role of "rites of passage" (Van Gennep, 1969), in the sense that not only do they indicate a change of didactic activity, but also confer a new

² The didactical means to get a student to enter into such a contract is devolution. It is not a pedagogical device, because it depends in an essential way on the content. It consists of putting the student into a relationship with a milieu from which the teacher is able to exclude herself, at least partially (adidactical situation).

status upon the pupil: that of a pupil who “does” maths. Here we find the definition of the rite as an act of institution in the sense that “it *signifies* to someone his or her identity, but in the sense both that it expresses it to him or her and that it imposes it on him or her in front of everyone [...] by notifying with authority what he or she is and what he or she is to be” (Bourdieu, 1982, 60). Such a pragmatic view leads us to study rites as acts that are not confined to talking but also involve producing or doing. In the same way that Austin (1970) shows how to do things with words, the performative dimension of the ritual should be questioned: beyond their existence – incidentally often forgotten as such – beyond their expressive and symbolic dimension, what do rituals do and what do they produce?

Protodidactic rituals

We employ the term “protodidactic” in reference to the term “protomathematics” used by Chevillard (1991), but giving it a different meaning: here we are talking about primitive and rudimentary situations that are introductory to the lesson and not necessarily in direct relation with that lesson. These situations interest us less in their didactic aspect than in their symbolic dimension, as creators of the conditions for entering the didactic situations. Indeed, it is in these brief but highly codified moments that we find the best illustrations of the symbolic and pragmatic function of the rituals. Here are two examples of this:

Class 1: Each mathematics lesson invariably begins by a round of “The numbers game”.³ A pupil draws 6 cards at random, and the teacher puts them up above the board. On the board, the teacher writes a number between 100 and 999, then starts the clock. The pupils get to work. After a minute, the teacher stops them. A pupil who has found the solution (or who is nearest to it) comes up to the board to show his or her calculations. The solution will be accepted or rejected without any further comment. Sometimes a second solution is proposed. Throughout this activity, very little is said.

Class 2: Testing of the multiplication tables is subject to very formalised practice: “*Take a sheet of paper, we’re going to do a test of the multiplication tables*”. On a sheet of paper, the pupils write the date and the title. The teacher then asks: “*OK, is everyone ready, has everyone prepared 10 short lines? Ready? Go!*” The pupils are focused, with their pens in their hand and ready to write. The teacher then gives them 10 multiplications to do at high speed. The pupils write the answers as the exercise progresses. Not a word is said (those who don’t know the answer or for whom the exercise is going too fast, put a cross instead of the answer). “*Stop! Pencils down! Collect the papers*”. Nothing

³ This is a part of a daily TV game show: “Les chiffres et les lettres”. Six cards are drawn at random. The values of the cards are as follows: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 25, 50, 75 and 100. The game consists of achieving or getting close to a number drawn at random between 100 and 999, using the four elementary arithmetic operations. Each card can only be used once.

more is said. They move on to the next activity. Words and actions are reduced to a minimum. The exercise runs perfectly smoothly and there are no interruptions or any later comments.

These observations give a good illustration of the words of Lévi-Strauss (1971, 600) for whom, in rituals, “gestures and objects intervene *in loco verbi*, to replace words. [...] The movements made and the objects handled are the means granted by the ritual to avoid speaking”. But although the rituals include few words, they “do” a lot and lead us to think that the symbolic function of protodidactic rituals counts at least as much as their strictly didactic function: although the use of elementary operations or the revision of the multiplication tables does have didactic interest, the creation of conditions that favour the introduction to mathematics lessons is essential to the definition of didactic situations. This is why, although they do not have a didactic function *stricto sensu*, didactic rituals nonetheless represent one of the means at the disposal of the teacher to make the changes of framework required to establish new teaching situations.

Progression of the lesson: frameworks of action

Like any regular, institutionalised social practice, the practice of teaching requires a number of regulated behaviours, repetitive actions, rituals or habits which provide a certain degree of predictability. As an element in didactic culture, ritual is habit-forming. A regulated ensemble of quasi-incorporated practices (actions, words), it is not, however, simply a passive appropriation of formal practices initiated or proposed by the teacher, since it contributes to the definition of the teaching situation and to structuring the action of the pupils.

Customs and the contract

The mathematics teaching situations that we have observed are not only rigorously divided up; they are often repeated and reproduced identically, irrespective of the knowledge in question and regardless of the teaching methods. Only outside disturbances (the arrival of a replacement teacher for example) can modify the habitual progression of the lesson:

Class 2: No classical lesson; individual work and personalised help. Activities listed on an individual photocopy (progress in stages towards a notion) are the only form of work, regardless of the nature of the knowledge under consideration (numeration, operations, problems, geometry, etc.). Work is strictly individual and the only interactions are those that take place between pupil and teacher as part of personalised help, either requested by the pupil or offered by the teacher. The only group phase is that of institutionalisation. During the replacement period, we will see a slow disintegration of the didactic rituals –in particular requests for help– ending up with an abandonment of individualised work in favour of lecturing type teaching.

Class 3: No lecturing but individual work taking place following a strict order: instructions, individual work on a photocopy, correction as a group. In 8 sessions (2 weeks), the duration of maths lessons was always 50 minutes. With the replacement, this clear organisation was broken both in the succession of activities and in the duration of lessons (35 to 70 min.). These disturbances have didactic consequences: the pupils do not recognise the situation (*What do we have to do?, What are we supposed to do?*), contest the didactic choices of the replacement teacher (*That's not the way we do it with Miss*) or refuse any participation or offer minimum commitment to a situation. The teacher is thus obliged to lower his expectations and to abandon his initial didactic choices.

The great stability of organisation of the didactic situations and the ritualisation of teaching practices may legitimately lead us to consider the classroom as a customary society, in other words a society governed by “a set of compulsory practices [...] and ways of acting established by custom; most often implicitly” (Balacheff, 1988, 21). In principle, these rules, which Balacheff himself likens to the perennial rules of the didactic contract, are didactic in nature and contribute as such to the definition of the didactic contract. Therefore, any abandonment of the custom or any disturbance in ritual procedures necessarily has didactic consequences. This is why it may not be necessary to make a differentiation as clear-cut as Balacheff suggests between the global nature of the custom that regulates the social functioning of the class over time and the local nature of the didactic contract, which is included in a system of reciprocal obligations specific to the content of the knowledge under consideration (Brousseau, 1998). Indeed, in teaching practice, it is impossible to dissociate –if only for purely theoretical reasons– what falls into the anthropological field (customs, rituals) and what falls into the didactic field (the contract). On the contrary, it is in the intertwining and the reciprocal support of these two levels that the conditions for setting up and operating teaching situations and the rules of the didactic contract are set down.

Micro-rituals and establishing routine practices

In lessons, we can identify a number of micro-rituals which *apparently* function almost independently of the forms of the lesson or the knowledge taught. They come in the form of ordered, repetitive acts based on standardised behaviours to which a precise function is attributed. Requests for help are part of these ritual activities, but they draw their effectiveness from their repetitive, routine nature.

Class 2: In this class, it is no longer the action but the displacement of the pupil that marks a request for help. Sometimes asked to do so by the teacher, the pupils most often move of their own accord, generally after each of the exercises to be done (progression strictly programmed on the photocopy). The teacher is seated at a table covered with a large plasticized plaque enabling the pupils to do their exercises “live” or allowing the teacher to do her “remediation” (in her own words). In two sessions, she gave help 107 times (4 to 12 times per pupil, or an average of more than 7 times per pupil). The regulation of pupils occurs

spontaneously: there are never more than 2 or 3 of them at a time by the teacher and they only come over when she is free. The replacement teacher, who attempts to continue with this organisation method, is soon bogged down (up to 9 pupils standing in noisy, agitated queues) and cannot adequately perform the didactic management of individual help.

Class 3: In this class in which the teaching is highly individualised, “*the pupils who don’t know*” call the teacher by raising their hand: “*I’ll come and see those who raise their hand*”; “*Those who have difficulties please raise your hand so that I can come and see what’s wrong*”. The teacher spends most of her time individually helping the pupils in response to their requests. Of course, such a ritual is not a totally regulated ceremony and leaves room for individual strategies: reinforcement of their call for help (calling out verbally); economic strategies (raising their hand when the teacher is nearby or when she starts moving around, and lowering it if she doesn’t come); avoidance strategies (not calling the teacher so as not to have to explain themselves)⁴. However, this ritual disappears very quickly with the replacement teacher, who does not answer calls for help. As early as the second day of the replacement, the pupils are no longer raising their hand to ask for help.

By creating habits and making practices routine, rituals create the conditions for didactic action. But as soon as they are no longer borne by an authority in control of their ceremonial progression, they lose their clear organisation and become no more than a banal activity without any didactic interest (class 1); and if the effectiveness of the action disappears, then so does the ritual (class 3). This is in no way a judgement on the didactic validity of such and such a ritualised activity, but emphasises the role of habit-forming actions and the stability of frameworks of action in teaching practice. Here we concur with the works of Voigt (1985) on the study of interactions in mathematics classes and on patterns of experience (which enable the subject to make an immediate definition of the situation, and to reduce the complexity), and more particularly his comments on the function of routines in didactic situations.

Didactic memory and involvement of the pupil

Didactic memory (Centeno, 1995) is one of the fundamental dimensions of any teaching activity. It contributes to the set-up of a shared didactic culture which is one of the bases of the didactic action of the teacher. It is on the basis of this founding of knowledge and common repertoires that the pupils are able to construct shared meanings and commit themselves to didactic situations. However, things are not always so simple. The scene below illustrates the difficulties encountered by a pupil in interpreting correctly the situation proposed by the replacement teacher of class 1 in a lesson devoted to the properties of circles. The lesson starts as follows:

⁴ Once can find a famous analysis of these teacher-pupil adjustments in Mac Dermott (1976).

Teacher: *I'm going to show you an object which is almost an antique now [she holds up a vinyl record]... In geometry, what do we call this object?*

Several pupils answer spontaneously but the teacher asks them to put up their hand, and puts the question to Camille.

Camille: *A disc!*

Teacher: *What about this?* [She moves her finger round the edge of the record].

Several pupils: *A circle!*

Teddy: *Who's the record by?*

A pupil: *We don't care!*

Several pupils [to echo this]: *No, we don't care!*

The lesson continues with the properties of the circle, list of definitions (radius, diameter, circumference), the construction of circles, then goes as follows:

Teacher: *Watch! I'll put a point here [inside a circle]... Is the point on the circle? ... Where is the point?*

Several pupils: *Inside it!*

Teacher: *What do you call all the points inside the circle?*

Marina: *The disc!*

Teacher: *That's right, all the points inside a circle form what is called a disc.*

Teddy: *Ah! So that's why you brought one in!*

Teddy's final exclamation confirms his incorrect initial interpretation of the disc being used for musical purposes. While the other pupils easily saw the didactic intention of the teacher (expressed when they say "we don't care"), it is only at the end of the session that Teddy makes the switch of framework required to understand the didactic action of the teacher and establishes the link between the object presented and the mathematic knowledge targeted. By introducing a new didactic process (showing an object) which has never been used before by the usual teacher, the replacement puts certain pupils in a conflict of frameworks: what should have been a didactic aid has become an obstacle to the set-up of the didactic framework. Should we therefore conclude that any didactic innovation should be proscribed? It is true that by guaranteeing the stability of teaching forms and by focusing attention on the essential, in other words the situation and the knowledge, the ritualising of teaching activities certainly facilitates the commitment of the pupil to the situation. However, does not the ritualising of school life, precisely because it involves repetition, restarting identically, and an emphasis on the formal, risk leading to a uniformity of action and a paralysis of the cognitive processes used in learning?

Conclusion

The observations presented above lie at the junction between the anthropological and didactic fields. Although they do not present major didactic issues, rituals still play an important role in the didactic organisation of teaching situations. We can consider that although some of the phenomena observed are not specific to mathematics (rituals or markers introducing the lesson, micro-rituals for requesting help, changes of framework), they nonetheless take particular forms according to the knowledge under consideration and contribute to defining situations and structuring the action of pupils in the framework of the teaching of mathematics.

It can also be considered, with Voigt, that routines are frameworks of action that are well adapted to certain didactic methods for the teaching of mathematics: “Mathematics education showing a rhythm of actions and expectations which can be rather precisely predicted is a productive ground for routines. The chance of acquiring routines, and of reproducing them in action, is particularly high in situations in which the active subjects must attain predetermined results, in which obtaining and processing information in order to make well-founded decisions is only at a high cost” (1985, 109). This is the first question: does the epistemological structuring process specific to mathematics lead to the strengthening of ritual phenomena, to the formalisation of didactic situations and to the framing of pupil action, in other words to a certain way of doing mathematics? Or conversely, is strong ritualisation not likely to reinforce the strictly algorithmic dimension of mathematics activity and to become an obstacle to proper mathematics education?

This leads us, in didactic terms, to pose the question of the place of these ritual practices in mathematics teaching situations: should we, for example, encourage the set-up of rituals, as has been the case in French infant schools, at the risk of straying into excess and creating a mockery of rituals devoid of any didactic virtues⁵? Or should we, on the contrary, identify more precisely these practices in order to control their effects on the school experience of pupils and their commitment to the task in hand? This naturally leads us to the question of taking these phenomena into account in the field of teacher training. But although we have outlined the potential risks to the learning process of excessive formalisation of frameworks of action, we should also question whether the ritualising and routine-establishing processes in mathematics teaching is not an obstacle to change for the teachers themselves, thereby making innovations in this field more difficult.

The question of identifying changes of framework is, in our view, of capital importance. In previous works (Marchive, 2003) we have shown the difficulties experienced by certain young pupils in their first year of compulsory schooling (6 years) in interpreting the rules and in switching frameworks in order to commit

⁵ For a study of the functions of rituals in infant schools, see Amigues in Amigues and Zerbatou-Poudou (2000)

themselves to the activity. The error of framework made by Teddy, too briefly reported above, is just one example of the problems that can be posed by the different methods used in didactic situations. It is thus necessary to find out exactly what, in these difficulties of framework, is caused by the very structure of mathematic knowledge (formalisation, algorithmic dimension ...): within the didactic situations themselves, does the pupil not have to operate multiple switches of framework with regard to the use of mathematic knowledge, thus forming obstacles to learning?

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THE TRANSITION FROM INITIAL TRAINING TO IMMERSION IN PRACTICE. THE CASE OF A MATHEMATICS PRIMARY TEACHER

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Abstract: *Our objective is to know and understand professional development, centred on reflection, of a teacher in the period between the final stage of her initial training and her first experiences as a primary teacher. This work is centred on the information collected through practice memoirs and from the classroom recordings from her first year and have been analysed according to grounded theory. We'll see how the very close relationship which she has maintained with the educational context stamps some specific characteristics onto her professional development. We'll analyse the interaction between our initial conceptualisation of professional development and the observed case.*

Keywords: professional development, reflection, novice primary teacher, pre and in-service teacher education.

The present study is part of a collaborative research project whose objective is to know and understand the professional development of a novice primary teacher in relation to mathematics teaching. This investigation collects information throughout her first two years as a teacher in a primary education centre, as well as from the teaching practice carried out during the last year of her degree. Here we present the beginning of this process: the last part of the degree and the first months as a professional. Through this we will attempt to get to know the departure point of our teacher (Julia) before taking part in the aforementioned project. It must not be understood as a static photograph of her professional development, but as a process in which her professional and personal experiences intervene, and it is in this sense that we assume the idea of evolution. This allows us to interpret her practice in the light of the reflections which she expresses as a student primary teacher.

Rationale

Starting from a conception of professional development as a process which is produced throughout life and integrates the stage of her initial training, we study the professional development of a primary mathematics teacher in the period alluded to.

Habitually, in the literature about professional development reference is made to the processes of change, improvement or growth, specifying these with aspects such as

knowledge, conceptions and practice. Climent (2002) groups these studies in three blocks. On the one hand, there are all those studies which consider that professional development comes defined as the closeness to a determined model (to the investigative model of the tendencies regarding the conceptions with respect to the learning and teaching of mathematics, for example – Carrillo & Contreras, 1994). Other types of studies focus on knowledge to explain professional development and they consider that this is determined by the learning of the mathematics contents themselves as much as by the pedagogical content knowledge. Finally, other investigations understand the professional development of the teacher in a global way, considering conceptions, the knowledge and the practice in an integrated manner. In this sense, we can cite Cooney (1998) (whose key idea is reflective and adaptive practice), Krainer, (comprehension of practice, 1999), and Jaworski, (reflective practice, 1998), which emphasises reflection as a fundamental motor of development and which serves as a base for the conceptualisation of professional development which Climent (2002) sets out. As a consequence of the observation of the development of the primary teacher in her research, Climent considers that the major issue is the closeness to practice from a more complex perspective¹. The author associates the development to a “progressive taking into consideration of the complexity of the practice and of the learning of the pupils, and the analysis of this and the performance considering more and more elements and adapting it to the learning of the specific pupils. It would be...a process of continuous learning as a reflective professional and criticism of her practice (concerning mathematics teaching)” (p.119). We shall attempt to show this conceptualisation is not only a theoretical element which serves to explain development but also a contributor to promoting it.

We have assumed the definition which Climent proposes (for experienced teachers) to describe and understand the professional development of our novice primary teacher in the transition period between her initial training and her immersion in practice. One of the key aspects of this definition is reflection. We are aware that the fact that, unlike ours, the teacher in Climent’s study was an expert, which can imply changes in this conceptualisation. Our goal is to observe to what extent this definition helps us to understand our object of study, or if, on the contrary, an adaptation is necessary and to contribute in this way, to completing it. We think, furthermore, that the analysis of Julia’s reflection during her practical period could be a good way of understanding many of her decisions and of her contributions afterwards in practice because in it the language and repertoires that she uses to describe her reality and her own experience, her interpretive systems and theories, and the context in which she

¹ Climent’s research focuses on an experienced mathematics Primary teacher’s professional development. She, Ana, took part in a collaborative project (PIC) on problem solving, with 2 researchers and other 2 experienced teachers. In the fourth year Julia started to participate in PIC, as well as the third researcher. We meet every 2 weeks and deal with discussion of documents, teachers’ observations and analysis, tasks designing and analysis of curricular materials.

works, are reflected (Schön, 1983). We are going to see, therefore, what relation exists between professional development and the reflection which we are concerned with.

We coincide with the authors who emphasise in the concept of reflection the relation between this and the resolution of problems, considering it as a way of facing up to and responding to the problems (Santos, 2000; Zeichner, 1993). In this sense, Jaworski (1998) considers a problematic situation as the origin of reflective practice, relating reflection with action. It addresses an idea of reflection orientated to change (Saraiva & Ponte, 2003); change in the doing, saying or thinking of the teacher. Through this reflection, the teachers become aware of their conceptions and aspects which characterise their own practice (making them explicit in such a way which can be objected to critical scrutiny, Jaworski, 1998, Schön, 1983, 1987), improving their understanding of the practice and of themselves with relation to this.

Methodology

Our principle objective is to characterise and describe the process of the professional development of a primary teacher from when she finalises her initial training to when she teaches mathematics for the first time. Specifically, we will centre on:

- Knowing the aspects on which a primary teacher student's reflection turns when she observes the practice of another teacher and seeing the differences which exist with the reflection on her own practice.

- Knowing and identifying the influence which her previous experiences have on her professional development.

Given that our objective is to understand in depth a singular reality, the method adopted is that of the case study (Stake, 2000). We hope through this to be able to generate hypothesis and discoveries. We subscribe, coinciding with the interpretive paradigm, to a relativist perspective of reality, considering that people act in the real world according to the meanings which they have for them, which comes up in the interaction with the objects and that it is developed through a process of interpretation. In this sense we want to move closer to the comprehension of Julia's meanings and interpretations with respect to the relative aspects of her practice (as far as the teaching of mathematics is concerned).

Our purpose has been to access to these interpretations trying to get the most out of the data themselves, looking to maintain an open posture to what these contribute to us, still aware that our professional as well as personal experiences and our professional knowledge, i.e., our *theoretical sensitivity* (Strauss and Corbin, 1994), would guide and enrich the investigating process.

To continue, we will explain the methodology which we have followed and which has permitted us to improve the comprehension of our case.

The field work started in her first year as a teacher (the moment she was first involved in the collaborative research project). To know and understand the departure point of our primary teacher, we recorded a didactic unit at the start of the course.

Our role was that of external observer, centring attention on the teacher, without intervening in the planning nor in the development of the sessions, from which information was collected. We realised that it would have been very interesting to have registered some type of information during the last few years of her initial training, with the objective of accessing the vision which she had of teaching, and to better understand her later immersion in practice. This information could be obtained through her *Practicum* report, which she had to carry out in the last year of primary teacher training. This work is the report which she had to produce during the practice period in a school. This is formed by three documents: in the first, Julia expresses her vision of the methodology of her practice tutor; in the second, she writes her reflections about each day's events, and in the last, she explains the design of her didactic unit to her putting into practice.

In the analysis of the data, we have analysed the *Practicum* report and then the video-recorded class observations. In the first case, we have combined the techniques of analysis of content (Bardin, 1996) with a flexible application of the method of constant comparison from Grounded Theory (Strauss and Corbin, 1998). We drew up two lists of categories which emerged during the process of analysis and which converted into a tool with a theoretic end, to help us to conceptualise the case.

The analysis of the recordings was based on the Schoenfeld et al. model (Schoenfeld, 2000; Zimmerlin & Nelson, 2000). We carried out a general scheme of the session, dividing them into episodes and sub-episodes (from the macro to the micro), after we analysed in detail each one of the chunks identified and, at the end, we made a summary of the more relevant aspects which had emerged from the analysis itself. We compiled a final report which made the typical features of Julia's practice clear.

To organise the information around the reflection we have used the instrument of reflection (Climent, 2002)². The familiarisation with this instruments increases our *theoretical sensitivity* making us more sensitive to identifying and inferring the relevant information from the data.

Julia studied teaching from 1998 to 2001 with the speciality of primary education. She was excellent and she showed a great interest in mathematics and its teaching. She has grown and lived the life of a school very closely as her parents are teachers. In this school, where she has always actively collaborated, she carried out her *Practicum* and started to work. Julia loves her profession and shows great dedication. From the first moment she has shown herself to be very interested in participating in the collaborative research project, seeing it as a great opportunity to continue learning with expert teachers and trainer-researchers. This case was especially interesting

² Understood as a series of aspects or dimensions about which it is interesting to notice in the analysis of the data: in which elements of her practice does the teacher centre her attention, what role does the mathematic content itself have in this reflection, the richness of nuances which she takes into account, in which moments she is reflecting, what consequences they extract and what worries reflect.

because it made the collection of information possible during the transition between the training period and her immersion in practice³.

Discussion

In this section we try to respond to the objectives we have set out.

Reflection on her tutor's practice

Most of the documents from the *Practicum* report are written from the perspective of the observer, so it was the role which she mainly performed in the first weeks of her practice. In them we can observe that Julia shows a greater sensibility towards classroom-dynamic aspects and the reactions and answers of the pupils. It seems that the pupils constitute her main source of learning because she allows them to know what they are like, what they think and how they react. This reflection on occasions is shown to be systematic, such is the case of the analysis of the following situation in the classroom:

It deals with the first time that the pupils come face to face with a subtraction with the units of the first number less than the second number (15-8). Julia classifies the types of answers obtained through an evaluation test and tries to reconstruct their reasoning. She differentiates the types:

15-8=10, because taking 8 from 5 leaves me with nothing (0) and 1 minus 0 is 1.

15-8=13, because 8 minus 5 is 3 and 1 minus 0 is 1.

15-8=3, because 8 minus 5 is 3 and 0 minus 1 is 0.

15-8=15, because you cannot take 8 from 15, so then the same is left.

15-8= 7 ...when I asked him [...] I came to the conclusion that he had tried to do it but as you cannot take 8 from 5 he decided to draw on his sheet in rough the 15 sticks and take away the corresponding 8. As we can see, this reasoning is the most logical and the only one that gives the true result, but it clearly shows at the same time how undisciplined the boy is as my mother is fed up with saying that you have to take away units from units and tens from tens (PD.40, 41)

The above-mentioned reflection does not appear to be used by her to extract consequences for her future action. She does not question her model of performance, implicit in the previous unit, when she underlines the pupils not following the norms given in the classroom. In the case of the experienced teacher (Ana) from our previous works (Climent & Carrillo, 2002; Climent, 2002), the analysis of the difficulties of the pupils was the departure point for thinking about the specific strategies of performance to deal with these difficulties. On that occasion the teacher

³ On the other hand, Ana is 20 year-experienced. Her branch was Spanish and French language, but she always worked as a generalist primary teacher. She is very concerned with her training, specially with improving mathematics teaching. She usually takes part in innovative courses and groups with other teachers.

goes deeply into the pedagogic content knowledge concerning how the pupils learn and how to teach. In the case of Julia, the analysis of the difficulties of the pupils appears to be an end in itself: to expand personal knowledge with respect to learning, without having repercussions in what she does as a professional.

Julia's position is interesting regarding the practice carried out by her mother as practice tutor. One feature which Blanco and Borralho (1999) stressed about student teachers is that until they get to teaching practice (*Practicum*) they cannot problematise teaching, fully trusting their capacities. This problematisation is produced when they face the more or less implicit model of teaching which the pupils bring to their practice with those of the tutor, which causes them to question their positions and to value the adequacy of the tutor's decisions. In the Julia's case it was not like this, her implicit model did not clash with her tutor's. All description about her tutor's methodology is expressed in a positive sense, with the exception of two explicit criticisms, with regards to group work and the use of manipulative resources.

The greater part of Julia's reflections in her *Practicum* report are after the mathematics class; an aspect which is normal because her role at this moment is that of observer. There are few indications of her adapting what she had planned to what happened in the classroom. The decisions which she takes hardly ever change the collection of activities planned. In this sense her plans of action are rather rigid (coinciding with what Zimmerlin & Nelson (2000) observe in a novice teacher).

Reflection on her own practice

In the document where Julia explains the process from the designing to the implementation of her teaching unit, she transfers the focus of reflection towards her own practice; she centres her attention on aspects linked with its implementation, reflecting principally on the sequence of activities and decisions which she takes. This idea coincides, in part, with Fuller and Bown, (1975, in Brown & Borko, 1992), which affirmed that novice teachers usually centred more on their own performance than on the learning of the pupils, considering it less problematic than their performance. However, when she adopts the role of observer, Julia centres on the pupils' learning.

By centring on her own practice (when it is she who teaches a teaching unit), we could expect that it will include certain criticisms, as she declared, "I am going to describe my didactic unit now, how I planned it, how it was put into practice, and what things it seems I could change," [PU.1⁴]. However, no reflection exists afterwards in which she shows her dissatisfaction with the decisions taken and extracts consequences for her future professional practice. On the contrary, she describes contributions in detail in which her explanations enable her pupils to

⁴ "PU.i" stands for the unit of information number "i" of the part of the *Practicum* report where Julia explains the planning and implementation of the teaching unit which she brings to the classroom. "PD.i" is analogue, now with respect to her *Practicum* diary (of the *Practicum* report).

understand and overcome their difficulties, demonstrating her good practice as a professional:

“When I go for a seven-sided shape, totally irregular, I say, ‘this is indeed a triangle,’[...] they all start to tell me it’s not, that I am making a mistake and [...] they all start to laugh. This learning strategy seems very interesting to me because they learn from my mistakes and furthermore it is tremendously useful because in this way you know if the attention of the class is being kept and the children find it very funny that the teacher makes mistakes” [PD. 66]

In addition, on some occasions she attributes the difficulties to the lack of attention to the students’ task and to its features, more than to the possible inadequacy in her methodological strategies:

“In some students also in the operations to solve from the evaluation they do a very curious thing, and this is that they subtract the units but afterwards they add the tens; I believe that this is fundamentally due to absentmindedness or because the children themselves are distracted.” [PD. 44]

In the case of Ana, coinciding with the idea of Fuller and Bown (op. cit), the teacher shows her certainty in her performance in the practice, which is why she focuses on the learning of the pupils. As far as Julia is concerned, she also appears to show she is rather sure of her performance. She appears to have assumed her mother’s model of performance; it is as if she had made her mother’s experience hers, without having lived it, assuming many routines and aspects of teaching in a way that converted them into tacit learning (Schön, 1983).

As far as her practice is concerned (in both periods, as student teacher and as teacher), it appears Julia does not reflect on mathematical content itself; she does not analyse didactically, valuing the difficulty which it might suppose for the pupils and the most adequate strategy of teaching for each moment. On the contrary, she only applies the same scheme of action for the teaching of whatever content, based on repetition: explanation, manipulation, completing the worksheet, correction and revision. In her planning she does not analyse the content to be dealt with either:

In the teaching unit of the *Practicum*, she works on the concept of the square as a shape with four equal sides, coinciding with the text book. In the activity of this the pupils are presented with more quadrilaterals, among them a rhombus (with unequal angles). She only points out the equality of the sides as definitive qualities of the square: *we’ll count shape by shape the sides...I say that all which have four [equal] sides we colour, because they are squares.* (PU. 78)

There is no questioning of what the book proposes. In the previous example Julia did not stop to think about the differences between both figures. Her previous reflection was about the design of the teaching unit, which consisted of selecting which activities from the book she was going to work on with the pupils and introduce small modifications to improve the limitations observed in her mother. The same happened in her practice as a teacher, with the difference that she selected the worksheets_of one day for the next.

Julia as a novice teacher

We believe that Julia's situation differs from that of novice teachers. Blanco and Borralho (1999) affirmed that the student teachers before teaching practice came more from the perspective of students than from their position of trainee teachers. Our case is different. It turns out to be significant that units of information exist in which she uses the first person plural to explain the characteristics of the centre:

“It seems interesting to me [this description] so that you can establish yourself... and know...how the school is like: norms, ideology and goals, discipline... so that you know...from where we start” [PD.3]

The fact that her parents had been primary teachers in this school for many years has permitted Julia from a very young age to have close contact with the life of the school. For this reason she describes the centre as a member of this community, which participates and shares norms and ideology. The same happens when she writes on her diary, in which she on many occasions also explains the decisions she takes as if they were fruit of the agreement between her and her mother. Her positioning in the practice is not of student, but already of teacher, with the difference that in the first documents she adopts the role of observer-teacher.

Although in the literature it is affirmed that one normally teaches in the same way that we were taught and that all our beliefs and pedagogic images are stable, implicit and resistant to change (Mellado et al., 1997), in the case of Julia it seems that it does not happen in this manner. From a temporal point of view, the time in which she was perceiving models of diverse teachers (her teachers) has been greater than the time perceiving her mother's model, and, nevertheless, it is this time which has exercised a more decisive influence on her vision of teaching and of learning, possibly because of the relationship which joins them, one which is so particular and unusual.

Final comments

In the case of Ana, her professional development had the feature of being more and more reflective, intervening more in the planning of the teaching, with greater flexibility to change what was planned during the course of the action and with reflections a posteriori channelled to changing her future practice. From Julia we would underline her capacity to reflect with care about her pupils learning (a posteriori reflections). These reflections do not at the moment appear to show potential to modify her practice. The reflection beforehand is limited to the selection of activities and it does not demonstrate flexibility during the course of the action. We cannot still, at the moment in which we find ourselves in this research, move forward on whether the conceptualisation of professional development through reflection will be useful for us in characterising Julia's development. But what is evident is that such conceptualisation allows us to approach and interpret this process, understanding it not in an isolated way but in the context of what has been already researched on professional development. With the continuation of Julia's immersion in the collaborative research project with expert teachers, we will see if her reflective

potential converts into a promoter of change, and if that reflection extends to her general practice (not only after the practice). Also we will be able to see if the referent of her mother, on clashing with the teaching model of these teachers, starts to contradict them, allowing the problematisation of her practice.

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PUPILS' ERROR TREATMENT BY CERTAIN HIGH SCHOOL TEACHERS, RELATING TO THE NOTION OF INVERSE FUNCTION

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Abstract: *This exploratory study proposes to consider the way that some high school teachers handle the errors made by their pupils, about the notion of inverse function. To study this handling, the research is focused on the teachers' accompanying speech when they correct their pupils in different created situations, precisely using the square root function. This study uses a questionnaire to obtain the teachers' accompanying speech (recollected in a written form), which is evaluated with a contents analysis. The work mainly makes reference to three theoretical aspects: (a) the speech role and its use in the pupils' learning, (b) teachers' representations about learning and teaching practices and (c) their personal relationship to the concept of inverse function.*

Keywords: Pupils' error treatment – inverse function – teachers' accompanying speech – teachers' personal relationship to the concept – teachers' representations.

THEORETICAL FRAMEWORK

Artigue (2003) and Bloch (2002) brought to our attention the influence of the evolution of the *knowledge to be taught* in the school programs in France, which is present even in the *institutional relationship of teaching institutions* and in the *personal relationship of teachers* to the concept of inverse function. These concepts are taken from the Anthropological Theory of Didactic (Chevallard, 1992). Inside the established universe, everything is considered to be an object, which exists as soon as a person or an institution recognizes its existence and makes it significant, at least for one of them. It is in this case that Chevallard introduces the definition of the *personal relationship of this person*, or the *institutional relationship of this institution*, to the specific object.

Moreover, Antibii and Brousseau (2002) separate the didactic transposition into two phenomenons: *effective didactic transposition* and *de-transposition*. The effective didactic transposition is made up of « *at least two kinds of transposition: the external transposition to the educative system, between the 'erudite knowledge' and the 'knowledge to be taught', and the internal transposition, between the 'knowledge to be taught' and the 'taught knowledge'* » (Antibi and Brousseau, 2002 [own translation]). They put the phenomenon of de-transposition in the *taught knowledge* level, where the teachers transform the previous taught knowledge into another closer to the erudite knowledge. The result of the whole didactic transposition process involves the effective

didactic transposition and the de-transposition.

With the teaching-learning process there are several representations. In this case, we will work only with some teachers' representations and for this, we will apply the same meaning to the word "representation" as did Rousset-Bert (definition provided by Abric in 1987): « *the representation is the mental activity process and product by which one person or a group of persons recreate the reality that they have in front of them and give it a specific meaning* » (Rousset-Bert, 1991 [own translation]).

On the other hand, when teachers talk with their pupils, especially in the classroom, they mix the specific mathematical speech with a general accompanying speech, which helps to support the pupils learning. Some teachers think that this accompanying speech can be useful « *to help pupils with the follow up of the mathematical speech, their local comprehension, their retention, and the knowledge to know how to re-use the studied content later* » (Chiocca, 1995 [own translation]). The author presents three roles in the teachers' accompanying speech, which help pupils in the mathematical learning processes:

- The communication role between the teacher and pupils seeks to have the pupils involved themselves and follow the mathematical speech. It is also used to motivate pupils in any academic situation.
- The structural role permits the establishment of the "mental organization" of knowledge. Teachers give points of reference to pupils to create links between certain content (using time or memory), give the order of the procedures to be followed in the exercise, reconsider other concepts previously studied, or evaluate the pupils' answer without giving complementary comments.
- The reflection role must facilitate pupils' comprehension. For example, teachers would give explanations and comments to make pupils take part in the content development. It is also possible to make an evaluation with other comments to enhance the observation.

To link these theoretical elements, we will describe some of the statements. The teachers' personal relationship to the concept is influenced, among other things, by the external didactic transposition, which establishes the knowledge to be taught. At the same time, the teachers' personal relationship to the concept will affect the remainder of the didactic transposition process. We can also say that in this structure of teaching practices, teachers' representations and teachers' personal relationship to the concept can be found in implicit ways in the teachers' speech. At the moment of classifying the teachers' accompanying speech in its roles, we would expect to find indicators of teachers' representations when there is a communication role or in different evaluations about the pupils' answers. However, the teachers' personal relationship to the concept could be more easily found when we identify the structural or reflection roles, because

the meaning of their contributions is closer to what the concept involves.

DESCRIPTION OF THE PROBLEM

This work will focus its interest in the notion of inverse function, particularly at the high school level, using the square root function. According to several authors, for example Bronner (1997) and Rousset-Bert (1991), the notion of inverse function appears in here for the first time, even if it is treated with an implicit character.

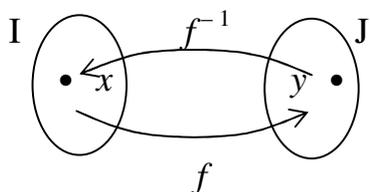
First, we are going to define our mathematical concept. We will understand the concept of *inverse function* in the following way:

When we have a function $f: I \rightarrow J$, which makes it correspond to any element x of I an element y of J , we can raise the question, “What are the conditions that allow us to affirm that there is another function g that makes y correspond to x (“the return”, named *inverse function*)?”

Then, a function $f: I \rightarrow J$ admits an inverse function, if it satisfies certain conditions (to be *bijective*):

- It is *surjective*, i.e., any element y of J has at least an antecedent x by the function f .
- It is *injective*, i.e., any element y of J has at most one antecedent x by the function f .

Thus, saying the inverse function $g: J \rightarrow I$ exists is equivalent to say that each element y of J has an antecedent and only one by the function f and it is denoted by $g = f^{-1}$,



so that

$$f^{-1}(f(x))=x \quad \forall x \in I$$

$$f(f^{-1}(y))=y \quad \forall y \in J$$

Our interest is focused on the teachers’ accompanying speech (which was distinguished above from the mathematical speech). So, from now on, we will refer to it simply as the teachers’ speech. In here, we will start to talk about pupils’ errors relating to the concept of inverse function, to analyze the influence of this speech in the pupils’ learning process. To easily define the different errors, we will use the three categories adopted by Rousset-Bert (1991) in her study about how teachers take into account the pupils’ errors relating to the square root:

DF (Définitions Faux): False Definitions. Definitions created by pupils that they make them work in connection with the square root. They can also be produced by the difficulty of the introduction of the inverse function, because the square root function corresponds to the first approach of the inverse function.

VA (VALidation): Validation. Validity field errors related with the sign and the inverse function concept. This category was gathered from two types of errors. The first one

concerns the definition set, the other one is the confusion between the validity set of \sqrt{a} and the convention that \sqrt{a} indicates a positive number.

AL (ALgébrique): Algebraic. Algebraic calculation is mistaken for the square root, which returns to errors in the operations.

It is important to mention that we adapted these three types of errors for this research and we do not work with the square root by itself, but as a particular example of inverse function. However, the VA error is not enough to cover all the errors concerning the definition sets for the bijective function, so it admits the inverse function. Because of that, we created a more specific type of error related to the inverse function concept: FR (Fonction Réciproque).

According to the previous theories, in relation to the object of this research and given our own approach, we have two main goals:

1. How do the teachers impart, in an explicit way, some of their own representations and relationships to the concept of inverse function in their speech when they correct pupils' errors?
2. What is the role of these types of speech in the pupils' learning and how do teachers use these types of speech?

Nevertheless these two questions are very general. So, we defined some small questions to guide the research:

- Is there an important utilization of the structural role in the teachers' speech at the moment of correcting the pupils' errors?
- Do the created situations give the teachers more opportunities to use the reflection role?
- Is the high teaching level a factor that indicates the necessity for the teachers to use the formal writing symbolic system in mathematics?
- If the teacher considers that the concept of inverse function begins with the exponential and the logarithmic, does he forget the square root function?
- The teachers who participate at an IREM (Research in Mathematical Education Institute) or who work as a pedagogical adviser, propose more activities to make links with the concept of inverse function?

METHODOLOGY

This study is centered neither on the mathematical content, nor on the pupils; it tries to approach the teaching practices. We wish to describe the effectiveness of the teachers' speech for the pupils' learning. Thus, the election to study this speech in this level is due, overall, to the influence that the teachers could have on the mental representations

and the personal relationships to the concept of inverse function with which pupils would arrive at the university.

The concept of inverse function is not a subject by itself in high school in France, but it is implicit in several mathematical teaching situations, resulting in not having a specific moment to observe a class. Thus, we made the choice to design a questionnaire based in a bibliographical study about pupils' errors, which was then proposed to a group of teachers. Of course, this situation created a limitation to the study, because the written speech and the questions about invented errors could generate somewhat unrealistic answers from the teachers.

The data gathering was carried out by the Internet without specific experimental design, from the statistical point of view. Thus we have worked with a sample available and our study is more of a case study than a statistical study.

The questionnaire includes three parts:

The first part will identify how the teachers could act and answer to different pupils' errors. The errors proposed relate to the concept of inverse function within a class situation. Each situation corresponds to each type of error. The teachers' answers allow us to identify some indicators that relate with the teachers' personal relationship to the concept of inverse function, and their representations about the teaching-learning process. Following is an example of a question from this part:

What would you say to a pupil of Second* who would make the following error? :

$$\ll \sqrt{x} = 4 \Rightarrow x = 8 \gg$$

This error corresponds to type DF, because it can be due to confusion between *double* and *square*. According to Bronner, it can be caused by an "absence" of inverse function.

The second part has questions specifically oriented to surface important aspects of the teachers' personal relationship mentioned before. Here following is an example:

Do you consider the use of the mathematical writing symbolic system necessary to develop the concept of inverse function?

It is possible that the teaching level has had an influence on the positive answer to this question, because the university teachers could emit the need for recourse to this register much more than the secondary teachers.

The third part aims to describe the sample available and it offers moreover the possibility of crossing some variables, trying to locate the differences, if they exist, relating to these variables. These include **sex, teaching experience, teaching levels in**

* A pupil of Second at Higt school in France is 15 or 16 years old.

which they teach, and their experience as a pedagogical adviser or in an IREM.

Concerning the classification of the answers in each of the speech roles, we will show several examples of teachers' speech to differentiate these roles.

An example of the communication role is when the teacher requests the pupil's opinion:

“Are you sure?”, “In your opinion...”, “Is $-3x$ negative?”.

Certain teachers could use the structural role to try to structure the pupils' knowledge calling upon other situations previously approached:

“It is necessary to return to the significance of square root”, “Remember, we have already spoken about it, which is the sign of...”.

The reflection role appears, for example, when teachers try to establish for pupils a justification by counter-example:

“We should not believe that $-x$ is always negative, we can find examples”;

or, to generalize or deduce something coming from the knowledge studied before:

“We can deduce from the solution of this equation, a property of the style...”.

RESULTS AND DISCUSSION

There were 18 teachers who participated in this study. The results show the way in which certain teachers would support the pupils' learning, because of their speech when they corrected their errors. The relevance of studying the teachers' speech is that it is a way of transmitting implicitly the teachers' personal relationship to the concept of inverse function, because it could have the objective that pupils create a personal relationship to the concept.

The principal interest of the first part of the questionnaire was to observe the role and its use in the teachers' speech when they react (in written form) to the pupils' answers. Thus, table 1 shows the summary of the teachers' answers in this part.

Table N°1

Classification of teachers' answers according to the role of their speech to facilitate the pupils' learning.

Error	Communication	Structural	Reflection
DF	3	12	7
AL	4	15	10
VA	7	11	18
FR	9	14	13

Results concerning the speech roles

Table 1 shows that the number of comments with a communication role increases according to the difficulty level of the exercise that supposedly was suggested to the pupil. For example, several teachers do not establish the link between the type AL or DF errors and the function concept, even less with the inverse function concept, but in the type VA or FR errors the relationship to the concept is more clear. This fact is reflected when teachers call the pupils' attention to the definition sets. Moreover, half of the teachers use the communication role in their speech when they correct the FR errors. From this, we could formulate the hypothesis that there is a rise in the communication role in the teachers' speech at the introduction of different concepts corresponding to the level of difficulty. This could be because pupils could need more interest or be more involved in the learning situation.

We also realize in the table 1 that the structural role is usually the most used in the pupils' errors correction. From this we formulated the hypothesis that it would constantly be used at the moment of correction of the pupils' errors. In fact, teachers will often foster that pupils remember concepts previously used, even if there are also a lot of evaluations without comments, which also has a place in the structural role. The evaluations with a structural role in the teachers' speech can be useful to the pupils in distinguishing what they already know of what they must revise.

Concerning the reflection role, the totality of the sample uses it in the VA error correction. The arguments most often mentioned are the explanation by counter-example and the checking of solutions, or the alternative, which is to seek the resolution field of the algebraic expression. Thus, we could formulate the hypothesis that teachers have a learning conception in which pupils must reflect to understand what they do, why they proceed in this way, and in which situations they can use the same reasoning. In this case, the objective of the reflection is that the pupils find the meaning and the understanding of their own answer or procedure, because even if they can partly be right, they could confuse, or not understand, the reason of their solution.

Finally, there are two general results in relation with the teachers' speech roles, which can be attributed to the tool used in the study. Firstly, few teachers use the communication role of the speech, because the instrument used to collect the information is written, and the communication role appears more easily when the speech is done orally. In addition, the reflection role is commonly used, because teachers had time to think of their answers and to think of a special situation where they could clearly express and amply their ideas.

Results concerning the kind of errors

With regard to the DF error, teachers mainly employ the structural role. The teachers' idea is above all to reconsider the concept of square root previously studied. This fact is

strongly related to the teachers' personal relationship to the concept of inverse function. It is possible that this error is perceived as confusion between addition and multiplication. In this case, we mentioned the problem of the introduction of the inverse function concept.

With the AL error, teachers typically use the structural role trying to establish a method to solve the proposed equation. However, the reflection role is also used several times. It is linked to the teachers' personal relationship to the concept of inverse function, because teachers desire that their pupils would individually develop the understanding process at the moment of solving the equation.

When the VA error appears, teachers clearly prefer the reflection role to find the cause of comprehension problems for the pupils. The most commonly used arguments are to promote the demonstration by counter-example, to find the definition sets, or to check the validity of the result.

Lastly, the FR error shows a similar use of structural and reflection roles. Teachers employ arguments with a structural role, when they want to establish a method to find the antecedents by the function, or to structure the pupils' answer. To become aware of the function's definition sets, teachers chose to use the reflection role.

Others results

Among the most important aspects of certain teachers' representations on the teaching-learning process, we found those that correspond particularly to the teaching practices. For example, they take on responsibility of showing pupils the exercise's solution:

“Take $x = -4$, $x^2 = (-4)^2 = 16$, you must have forgot it”;

They also tell them that the answer is false without making other remarks to try to find the errors:

“You are wrong, you have two possibilities”.

In the results of the second part of the questionnaire, we realize that teachers are aware that the inverse function concept is subjacent in several situations in Mathematics teaching. For example, 7 teachers, from the sample, affirm that the inverse function concept appears for the first time in a teaching situation in Second, and 6 teachers mentioned previous situations to the square root function. Thus, even if they are aware of the implicit link between the studied content and the concept of inverse function, they do not mention it to pupils, partly, because of the institution constraint and the teaching programs. That corresponds to the teachers' institutional relationship to the concept of inverse function.

The majority of those who answered the questionnaire do not consider the use of writing symbolic systems necessary to approach the concept. Besides, all of them state to

propose activities to find antecedents by a function with its graphical representation. However, the majority of the teachers affirm not linking this activity to the inverse function concept, which is implicit.

Thus, we could formulate the hypothesis that teachers clearly have their responsibility to take into account the teaching program. In general, they follow the proposals given in the material, without leaving much room for their personal relationship to the knowledge, and, they entrust the university with all of the responsibility to approach the inverse function concept.

CONCLUSION

Initially, the choice of working with teachers enabled us to perceive the personal relationship of certain teachers to the concept of inverse function, which is implicit in most of the content, according to them. This implicit relationship could be used explicitly at the moment when teachers transmit a message and they correct the pupils' errors. We could ask ourselves, as a research perspective, what is the pupils' personal relationship built in relation to the concept mentioned, knowing that it was not clarified in the content.

It is possible to interpret a largely implicit hypothesis in this study: the students' errors at the university are caused by the teachers' speech in high school. However, our possible implicit hypothesis has a different point of view, in the sense that the teachers' speech does not really support the pupils learning.

Another perspective of this research is going to study the inverse function object at university level. We could work at this level, with the teacher's effective speech in class, because the inverse function concept would be explicitly treated. This choice would test the hypothesis on the basis of written speech by adapting them to the oral speech used in the classroom. The work at the university level will be enriched by the results of the study of conceptions, which high school teachers could have transmitted to their pupils. Indeed, the de-transposition phenomenon could help us to look for the possible errors origins or the university students' difficulties. They could come from the pupils' personal relationship, built in high school, to the concept of inverse function.

This type of study could also propose other research more specific to the curriculum level, or many de-transpositions engineering to approach the inverse function concept at the university.

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EXPLORING THE ROLE OF VIRTUAL INTERACTIONS IN PRE-SERVICE MATHEMATICS TEACHER EDUCATION

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Abstract: *This paper analyses a virtual supervision setting, including e-mail and forum, during the practicum, in a pre-service secondary school mathematics teacher education program. It is a research about the authors' own professional practice that follows a qualitative-interpretative approach and is based on case studies of student teachers. The results show that the setting was significant for pre-service teachers with a more reflexive attitude but not to the others. The forum enabled fruitful reflections and discussions but the e-mail was mostly used just for organizative matters. Future attention must be paid to the role of the educational supervisor in establishing a culture of participation in the forum and of fluent use of the e-mail.*

Keywords: Preservice mathematics teacher education, Information and Communication Technology, Virtual interaction, Reflection.

In Portugal, the practicum is the last stage of secondary school mathematics pre-service teacher education. It is a practical experience in which the student teacher carries all the usual responsibilities of a teacher (with two classes) and attends seminars and other activities with the school mentors and university supervisors. This project aims to explore the possibilities of a virtual communication interface between supervisors and student teachers. The fundamental motivation to create this virtual supervising setting stands on the interest of the project team members, who are simultaneously teacher educators and researchers, of finding ways to shorten the “supervising distance” between the supervisor and the student teacher.

THE PRACTICUM AS A TEACHER EDUCATION STAGE

The young teacher faces countless problems. Hammond (2001), who studied teachers in the beginning of their careers, indicates that they point five key problematic issues: the attitude of pupils regarding the proposed tasks, pupils' misbehaviour, lack of support from colleagues, inadequate planning of classes, and bureaucracy. The practicum has two main resources to promote the ability to deal with such problems. One is the supervisors' support, specially the school mentor and the university supervisor, namely through clinical supervision. This kind of supervision pays a

central attention to the classroom and assumes the student teacher as the key participant in identifying, diagnosing and overcoming the difficulties with the help of the supervisor (Alarcão & Tavares, 1987). The other resource is the practicum¹ group and the other participants in the teacher education process (student teachers and supervisors from other practicum groups), working in a collegial style, stressing discussion, experimentation and critique. This is based on the idea that “teachers can not be self-sufficient” (Day, 2000, p. 110), but need each other’s support.

The potential of ICT (information and communication technology) as a working and communication tool for the teacher, notably as a support for the development of a new professional culture emphasizing virtual learning and sharing networks has been widely recognized (Ponte, 2000). For example, e-mail has been used as an element of the supervision setting in pre-service teacher education. Yildirim and Kiraz (1999) found that student teachers and supervisors use electronic mail, regarding it as an important communication tool, but with a highly variable level of use. Also, Souviney and Saferstein (1997) explored the possibilities of electronic communication in clinical supervision of student teachers and found that clinical messages exchanged between supervisors and student teachers could attain a remarkable weight within the e-mail correspondence (32% of messages).

The forum has also been used in pre-service teacher education. For example, in a study carried out by Heflich and Putney (2001), the discussion of professional issues lasted for eleven weeks in a restricted conversational space. They indicate a good level of argumentation but also a great variation in the number of interventions from the 22 student teachers (between 19 and 2 interventions each). Also, Bodzin and Park (2000) carried out a study using a public forum and conclude that student teachers’ discourse depends on their level of interest in the topic, its immediate relevance to each participant, and on interpersonal factors among participants.

This suggests that a virtual supervision setting, including e-mail communication and a forum involving student teachers and supervisors discussing experiences, problematic situations and puzzlements emerging from professional practice, has promising potential. However, the great variety of possible options regarding the objectives and working models of this kind of setting suggests that further empirical research is necessary, based on the realities and needs of each program and country.

THE VIRTUAL SUPERVISION SETTING

The present study concerns a supervision experience involving three practicum groups during 2003/04 with a virtual supervision setting, including a discussion forum and e-mail communication. This setting worked in parallel with the usual activities of the practicum, that include work in the school (preparing and teaching classes, reflecting about them, participating in school events and seminars), activities

¹ In Portuguese, “núcleo de estágio”. Each practicum group usually includes two or three student teachers, the school mentor, the scientific supervisor (appointed by the Department of Mathematics) and the educational supervisor (appointed by the Department de Education).

with the scientific supervisor (preparation and presentation of mathematics topics), activities with the educational supervisor (reflecting about classes, discussing educational problems, doing an educational project) and activities promoted by the university with all students teachers, mentors, and supervisors. With the virtual supervision setting we aimed to provide a more permanent support to all student teachers and favour their development of professional practical thinking, reflexive and critical capacities, and attitudes favourable to collaboration and collegial work.

The virtual setting was constructed by the project team. The discussion forum and e-mail are viewed as teacher education resources with complementary potential. Both involve a written mode of communication. The forum enables sharing and discussing issues emerging from professional practice as well as wider educational questions within a broad group of student teachers and supervisors. The e-mail aims to strengthen the critical, reflective and communicational dimensions of the supervision process. Such communication, contrarily to the forum, is restricted to the practicum group or to some of its members. It is the student teacher that must ultimately decide if a given issue must be raised in the forum (wide discussion), sent by e-mail (restricted discussion) or discussed by any other mean. The forum had two phases, one from November to February and another from March to May, and was moderated in each phase by two members of the project team. In order to facilitate the participation of student teachers in the forum some themes were established. Also, to clarify the expectations regarding contributions to the forum and sending messages by e-mail, some rules were established. Student teachers were informed that their performance in this experience would constitute an element for their evaluation of the practicum. However, it was established that participation in the activities of this virtual setting would not replace the development of an extended educational project.

To attain a better perception of the practicum and the virtual supervision setting, a face-to-face meeting with the participants in this activity was held half way through the school year. This meeting had two main points. First, student teachers from each practicum group presented an extended reflection about an activity carried out in the practicum related to pupils' assessment. This topic was selected because it was widely discussed in the forum. Second, they provided a brief perspective about the development, so far, of the virtual supervision setting. This meeting was held with the participation of all student teachers, school mentors, educational supervisors and members of the project team.

RESEARCH METHODOLOGY

This study follows a qualitative-interpretative approach. The participants are student teachers, in the 5th year of the mathematics teaching degree of the Faculdade de Ciências da Universidade de Lisboa from three different practicum groups². Six case studies were carried out, one of each student teacher, taking into account his or her practicum group. The educational supervisors of these groups (Hélia Oliveira, José

² All the names of student teachers, school mentors and scientific supervisors are fictitious.

Manuel Varandas and Paulo Oliveira) belong to this project team. Given the ties that the virtual supervision setting has with the professional practice of the members of the team and the fact that the work developed in this setting is part of the problem under study, this research constitutes an investigation about the authors' own professional practice (Ponte, 2002). Data were collected through two semi-structured interviews, one carried out at the beginning and the other at the end of the study. Furthermore, the messages sent by e-mail and to the forum by student teachers and educational supervisors were also used in the analysis.

PERSPECTIVES ABOUT THE VIRTUAL SUPERVISION SETTING

Using e-mail

In their interviews, student teachers refer mostly to the forum, ascribing e-mail a secondary role. This may happen because participation in the forum was compulsive (two messages were required every two weeks), whereas communication by e-mail was not. Therefore, e-mail was mostly used to set up meetings, organize activities, send working documents and, in some cases, clarify specific questions.

There was a range of different ways student teachers related to e-mail. One of them, Francisco, is a strong user of the Internet since the 4th year of his university program, doing searches and communicating with friends. During the practicum, he used e-mail a lot to contact the educational supervisor – he sent 19 messages, just by himself or with his partner Rogério, an average of 3 messages a month. When the message includes his partner the subject tends to be general, and when he is the single sender it tends to address specific issues. By the end of the school year, when he and his partner were publishing their educational project on the Web, they used mostly the cellular phone and sms messages to contact the supervisor because they expected a faster reply. It should be noted that during the practicum he and Rogério were usually connected at home through the Messenger, exchanging ideas and files.

Another student teacher, Sílvia, used e-mail in the previous year, namely to contact with her university teachers. She feels that this is a good way to send them papers, to ask questions, or to solve small problems. During the practicum, she used e-mail now and then to communicate and send materials to other student teachers. With her educational supervisor, she used e-mail to send lesson plans and reflections regarding her educational project, as well as to arrange meetings and clarify details regarding the classes to be observed. She has no problem in using e-mail to solve minor problems but she does not see it as a good means to reflect about complex and important issues – for those she prefers a face-to-face conversation.

A third student teacher, Alda, only uses e-mail “if it is absolutely necessary”. To speak with other people she prefers to use the telephone or to meet in person, as “machines are very impersonal”. For her, e-mail is efficient to contact university teachers and supervisors, but “it is not a way to speak with friends”. In the beginning of the practicum she had access to the Internet at home, but after January that was no longer the case. However, Alda only sent one message to the educational supervisor (24 April), to arrange details of a lesson. The few e-mail messages exchanged

between this practicum group and the educational supervisor were sent by her partner Carlota. However, Carlota does not like this form of communication either – in her view, it presupposes a rather distant relationship. Therefore, in her case too, e-mail messages only concerned procedural and routine matters.

All student teachers agree that e-mail interactions must not replace face-to-face interactions in visits to schools. In fact, in this experience, that was far from happening. E-mail was mostly used to help organizing activities and sending documents. It remains to be seen (i) whether student teachers can develop some willingness to write messages regarding professional problems, and (ii) knowing how to overcome the difficulties in establishing a closer relationship between student teachers and educational supervisor. These two conditions seem necessary so that e-mail may have a more significant role in clinical support to student teachers, enabling the educational supervisor to maintain a shared reflection with the trainee when a face-to-face contact is impossible. Another question is (iii) whether e-mail can be replaced in that role of clinical support by software such as the Messenger.

Using the forum

As we stated, the forum was designed to discuss questions of common interest to the student teachers. It began (Nov./Feb.) with three themes (Critical incidents, General educational issues, and Post-lesson reflection) and, in a second phase (Mar./May), just two themes (issues in General education and in Didactic of mathematics). Because of technical problems, the transition of one phase to the other, took about a month and a half. That, plus the accumulation of other tasks and the approaching of the end of the school year, led the second phase to have very little activity.

In the first phase, we noticed that very few messages had to do with specific mathematics education issues. Therefore, in the second phase, a new theme was established to promote reflections on problems related to teaching and learning mathematics. There were seven messages in this forum. Message 1 was a short welcome by a supervisor. Message 2 was a long (three pages) writing sent by Estela (Sílvia's partner) describing their initiative in setting up a mathematics laboratory in their school, aiming to motivate pupils towards mathematics through interesting activities and materials. She comments on their initial difficulty in attracting pupils to visit the room and then she describes how those who come got involved with high enthusiasm in the activities. She also asks herself why so many pupils have a negative view of mathematics. Message 3 was a reply by Francisco who commented the involvement of the pupils in the activities and related a similar experience in his school's annual mathematics week, which included a room with mathematical games. In Message 4, Francisco introduces a new topic, mathematical modelling. He discusses how this topic is considered in the secondary mathematics curriculum and proposes to address it in a more explicit way than what, in his view, it is usually done by teachers. Message 5 was sent by Sílvia who also introduced a new topic – mathematical essays – that she used in her grade 8 class, in the chapter in similarity of figures. She was pleased with the work of some pupils but sorry because others

had not done it. Sílvia describes her strategy and the directions given to pupils and reflects on her difficulties, notably in evaluating the essays. Message 6 was sent by Rogério, describing an experience on using the Internet to do a statistics study. He reflects on the involvement of his pupils in this activity. Finally, message 7, was sent by one supervisor who praises Rogério's experience but asks two questions to understand better what he asked from pupils and what was final result.

Francisco provided quite a significant contribution to all forum themes. He sent a total of twelve messages (two of them together with Rogério), some as original contributions and some as reactions to issues raised by other participants. His first individual contribution took some time to appear and he speaks of that experience:

I remember that I had some difficulty in the beginning to send the first message. My problem was to find issues that I though were relevant for the forum. [...] What can I say that is relevant enough so that others may answer my question? I felt that problem many times...

In the second phase, he sent two contributions, one original and one reaction. He feels he participated less than he would have liked to.

Sílvia also refers that the construction of written texts was a difficult task, taking up a lot of time. It was necessary to start by choosing a theme that could be interesting to others. Besides, she had to program her participation in the forum so that she had the time to write a message:

At some point I had set up a schedule to reflect and to think on the theme. Because sometimes we have many themes, many things... [...] Sometimes took so much time to answer... In the meantime there were thousands of things that I would like to say but then I had difficulty in coordinating all the ideas so that they were not very confusing and that was difficult.

She feels that there was some limitation in her participation in the forum because she was afraid of exposing herself too much before her colleagues. She took a lot of care with what she wrote because of the image she might project and she feels that the same was true for the other participants. Asked if she was afraid that their messages were not well interpreted by others, she replied:

It was more because I felt exposed [...] The student teacher, at some point, draws a limit, isn't it? Up to a certain point we talk but after that, perhaps, it is better to stick to my practicum group, to my supervisor, isn't it?

Another student teacher, Carlota, refers a similar experience. For her, writing was rather uneasy, because the public exposition of its content. Like the previous student teacher, she indicates that her difficulty in writing was an inhibiting factor regarding her participation in the forum. Another serious difficulty that she felt was the lack of time: "As I had trouble in writing, then it took me a long time and then as I had no available time I had great trouble in participating". Her own personal and family life

also imposed restrictions on using her time and she ended up giving priority to the most urgent practicum activities and left behind her participation in the forum.

Her partner, Alda, states that her weak participation in the forum is also related to some inability to attain the aimed goals – “the goal was that [we problematize practice]. We couldn’t get there”. In her view, that happened because of the lack of experience of the student teachers: “One person is not [...] I mean, has no experience to understand some things and understand that they should be discussed”.

All the student teachers mention that they had difficulties in writing to the forum – choosing the topic, deciding on the content, finding the appropriate form, finding the time needed for this task. Three factors seem to contribute to those difficulties. First, the lack of fluency in the written language, as student teachers in general write little and often have an uneasy relationship with this form of communicating. Second, the lack of knowledge about this new communication space and the fear of being negatively evaluated, since writing to a forum is similar to speaking to a room full of people that we do not see. Third, student teachers still have little conceptual and discursive means to reflect about educational problems and classroom situations, as they are still at the beginning of their professional journeys.

Another problematic aspect of the forum was its dynamics. For example, Sílvia indicates that she was disappointed with the weak interaction that, in general, there was with the supervisors. Because she was permanently in touch with her school mentor, she had a high expectation regarding the feedback of her educational supervisor, given her broader experience and knowledge:

I expected perhaps more from the supervisors and the other teachers [...], that is, more answers... According to their experiences, according to their work, isn't it? From the student teachers I expected more “Oh, this also happened to me!” or “I did it this way or that way”, but not quite as an answer.

However, one must note that the dynamics of the forum varied with time. In the first phase, some themes had an interesting dynamic. The fact that in some periods the forum was not active with new messages had a demobilizing effect. Seeing nothing new for several times, led some participants eventually to give up.

During the year, the role of the supervisors in the forum was discussed several times by the project team. One perspective was that the forum should be essentially a space for the student teachers, where they could take the initiative to raise issues and comment each other’s ideas. The role of supervisors should be essentially regulatory, to be carried out as needed. Another perspective was that the supervisors’ role should be much more interventive, participating in the conversations, stating their position, raising new issues and suggesting new topics to discuss. The position officially adopted in the project assigned a rather active role to the supervisors. However, in practice, for several reasons, they had trouble in assuming that role.

Pre-service teachers' experiences

Notwithstanding his dissatisfaction with certain aspects of the forum, Francisco elects it as the stronger side of the virtual setting. He considers that the forum provided him with the best opportunity to raise and share his questions, doubts and experiences:

Although I know Sílvia, Estela, Carlota and Alda, perhaps, if I was at the college or in the public transports or at the coffee shop with them, I would never raise questions that really worry me in the practicum. Perhaps in that sense the forum wins points regarding the e-mail and direct contact because [it is] a space targeted for this kind of issues.

As a consequence of his enthusiastic relationship with ICT and his committed and reflexive attitude, Francisco had a very positive involvement in the virtual supervision setting. Despite the criticisms that he made, he thinks that, by and large, it was enriching to the practicum. His partner Rogério has a similar opinion.

Sílvia is also a very responsible and committed student teacher but, unlike Francisco, she has little enthusiasm in communicating through ICT. Her experience in the practicum, where she found many pupils unmotivated and even some with special education needs, was quite different from what she had foreseen. During the year, she had trouble in dealing with many situations. She used the forum to reflect on these issues, electing it as a privileged means to problematize her practice. Therefore, the forum provided her with a significant space for reflection. Even though its dynamics was far from what she would have liked, Sílvia says that she felt a certain emotional support from the forum and an incentive to continue to do all she could to help her pupils with their many problems. She refers that she gained from the contributions of her colleagues, as to how to act in some situations. In the final interview, she says that she finds it interesting to interact with more people and receive more opinions. Although she says to prefer face-to-face interaction, she has a positive image of the formative possibilities of virtual supervision. Her participation in this teacher education setting also helped her to develop a more favourable vision of the Internet.

Other student teacher, Alda, admits that ICT provides an efficient communication means, but she indicates her uneasiness, as she finds it “rather impersonal”. She claims to feel “not too well [...] somewhat lost. If I don't know what people are thinking about [...] I cannot find the proper words”. She recognizes that her use of the Internet increased with the practicum but relates that to a need to search for materials rather than on a need to communicate. For her, the logistical conditions of access to the Internet are determinant to allow or prevent a strong participation in this kind of setting. Alda regards her own participation as interesting up to January (8 messages) and non-existent from then on as she did not have a computer at home. In general terms, she considers that the virtual supervision setting was of “little significance” to her professional development. Even so, she considers that ICT promoted collaborative work among the several practicum groups, which would not have been possible in another way.

Her colleague Carlota also regards virtual interaction in an ambivalent way. She sees interesting potential for exchanging ideas, because of the variety of opinions:

I think that to have three [practicum groups] or so, ten [participants] or more, is a great advantage because there are more problems, there are more opinions, and it is a great advantage to me. As I only speak with Alda, I prefer to speak with more people rather than only with Alda, because Alda has one opinion and others have another one. Instead of getting one [opinion] I get three.

For Carlota, the virtual contacts between student teachers and university supervisors may improve communication but not their relationship: “it is another way of talking. It will improve a little bit [communication], it will. But not the relationship”. In her perspective, to have the opinion of a wider group of people regarding problems or episodes that she shared in the forum was not particularly useful. However, she recognizes that the participation in this virtual setting induced some collaborative work among the practicum groups involved:

Because even afterwards there were some colleagues who had [...] our e-mails and invited us to some activities that they carried out in the school and that in some cases we could attend. For example, with Francisco, who was close to our school, we participated in some activities. As they had our e-mail it was easier.

At the same time, Carlota felt that the work within her practicum group was somehow harmed because of the use of ICT: “Because I had less time to work together. I spent so much time writing the text that I lost time that I could use to work [...] with Alda”. This student teacher does not ascribe great relevance to the virtual setting.

CONCLUSION

The aim of shortening the distance between the educational supervisor and the student teachers was far from being attained with this experience. Even so, it yields interesting contributions towards reflection about the role of virtual interactions during the practicum. First, it is necessary to clarify what is sought with the setting. If the main aim is to strengthen the possibilities of clinical supervision, the main instrument that the educational supervisor needs to stress is an individualized communication means such as e-mail or Messenger. If the aim is to promote student teachers' reflective capacity through the development of a virtual community or a learning network, then it makes sense to stress the discussion forum. It is also necessary to reflect on the kind of didactical and evaluation contract that is established – is participation compulsive or optional, totally informal or used for the student teacher's evaluation? As with any other element of the supervision setting, the contract has strong implications in the way the activity is viewed by participants and in the learning experiences it yields.

Second, no matter what contract is established it should be noted that there are always barriers that need to be taken into account. As the cases of Alda and Carlota clearly

indicate, if access to the Internet is not easy and reliable, it is impossible to expect a strong involvement from student teachers in this kind of interaction. With the logistic problems overcome, there are other difficulties that arise such as the lack of time, uneasiness in writing contributions, as we see for the student teachers that participated in this study. We pointed out several factors that may contribute towards these difficulties and that require the development of writing fluency, familiarity with the new medium and the development of the capacity to analyse and reflect about professional problems. These are factors that the supervisor needs to pay attention to, and help overcome with his or her participation in virtual and face-to-face communication. The problematic activity of this setting shows that the supervisor needs to have a fundamental role, because, besides creating new opportunities of interaction, he or she needs to give an explicit and positive contribution so that they become effective and positive experiences of discussion and reflection.

The most committed and reflexive student teachers, evaluate in a positive way the experiences that they had with this virtual setting. The other student teachers' evaluation is not so positive, which seems to be related to their little commitment in the practicum. The challenge, here, is how to turn this kind of activity into a valuable learning experience for all participants, but that is an issue that goes much beyond the virtual supervision setting and concerns all the activity of the practicum.

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THE STUDENT TEACHER AND LEARNING OBJECTIVES: WITHIN A SOCIOCULTURAL ACTIVITY SYSTEM

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Abstract: *Learning objectives have become a feature of secondary mathematics lessons in England. In this paper we focus on one lesson in order to explore the potential of a particular framework for analysis. The language of a sociocultural activity system is used to review the place of learning objectives in student and student teacher learning in the context of a lesson and a lesson debrief. This is offered, not as a research paper, but as a philosophical consideration based on observation to begin to offer a way of interpreting some obstacles to teacher learning.*

Keywords: Socio-Cultural Activity Theory, Learning objectives, Pre-service teaching, Lesson analysis.

INTRODUCTION

In earlier papers we have considered a model that identifies the different elements which might come together and be transformed for student teacher's learning about teaching – professional traditions (curriculum, textbooks research etc.), practical wisdom (the activities chosen for lessons) and learner-knowledge (the knowledge needed to answer the questions). We argue that it is the interaction of these that lead to classroom events but it is only the sustained reflection on these that leads to the teacher-knowledge of mathematics (Prestage and Perks, 2001). We have continued our argument into a model for the knowledge needed for teacher educators (Prestage and Perks, 2003), i.e. university tutors and school mentors.

In this paper we explore aspects of this learning in relation to a sociocultural framework offered by Cole and Engeström (1993) and Engeström (1999). The framework allows us to consider the connections between the elements that influence the action of learning about teaching in the activity systems of schools and training partnerships.

In particular we analyse the role of a professional tradition, current in English schools, that of the use of 'learning objectives'. Introduced in September 2001, all public secondary schools in England have been working with what was originally called the National Numeracy Strategy, but renamed the Key Stage 3 Strategy, (DfEE, 2001) with pupils in the 11-14 age range. Learning objectives or learning outcomes have become a feature of mathematics lessons in secondary schools.

Askew (2004) in his analysis of the relationship between the objectives and the examples used in two primary lessons argues: “teachers subject matter knowledge for teaching will be central in mediating between these.” Subject matter knowledge for teaching is, however, an ambiguous phrase. We believe that it is important to unpick the many meanings ascribed to subject knowledge and Shulman’s (1986) pedagogic content knowledge, (Prestage and Perks, 2001). From our model Askew appears to use the lack of subject matter knowledge to mean the lack of learner-knowledge by suggesting that number connections were not available to the teacher. Our current thinking, based on our practice as teacher educators, is that the unsuccessful use of learning objectives may not be a function of inadequate learner-knowledge, but because like many professional traditions they exist as a rule to be followed not as a tool to expand learning.

Our practice involves many classroom observations; one lesson is offered here to illustrate our thinking. We have chosen to offer data from a lesson taught by a student teacher (ST) at the end of his professional year and the ensuing debrief with his mentor and university tutor. The data comes from observation notes, school documents and the lesson plan. These are described in terms from our original model with links to socio-cultural activity theory to counter the perspective offered by Askew (2004).

Ways of thinking about learning

In Vygotskian theory spontaneous concepts are developed through experience, by getting on and doing. Scientific concepts are developed through the context of instruction, through a deliberate pedagogic act

The development of spontaneous concepts goes from the phenomena upward toward generalisations. In the case of scientific thinking, the primary role is played by the initial verbal definition, which being applied systematically comes down to concrete phenomena Vygotsky 1986, p. 148

Scientific concepts are further characterised by being developed in a context of instruction monitored by a teacher. Though fundamentally different in nature, the development of scientific and spontaneous concepts represent two sides of the same concept formation:

Though scientific and spontaneous concepts develop in reverse directions, the two processes are closely connected. ... In working its slow way upward, an everyday concept clears a path for the scientific concept and its downward development. It creates a series of structures necessary for the evolution of a concept’s more primitive, elementary aspects, which gives it body and vitality. Scientific concepts, in turn, supply structures for the upward development of ... spontaneous concepts toward consciousness and deliberate use. Scientific concepts grow downwards through spontaneous concepts; spontaneous concepts grow upwards through scientific concepts. Vygotsky 1986, p.194

For Vygotsky, learning is mediated through the use of tools (such as language) and artefacts (lesson plans, debriefs). Tools for learning can be as diverse as the working in groups (Gomez and Rico, 2005), the use of e-mails and bulleting boards (Ponte et al., 2005) or the use and review of portfolios (Santos, 2005). Their importance, for us in teacher education, is how they enhance learning and the role in mediation by tutors.

Figure 1 offers an image for learning about teaching. The mentor (the subject of the interaction) is mediating her professional knowledge for the ST (the object of the mentor's actions). The tool here is the use of 'learning objectives'. By a mentor suggesting that learning objectives should be used, spontaneous learning may occur. However, unless there is some mediation, a deliberate pedagogic act, the ST's ideas are likely to stay 'primitive' becoming routine rather than demonstrating 'consciousness' and 'deliberate use'.

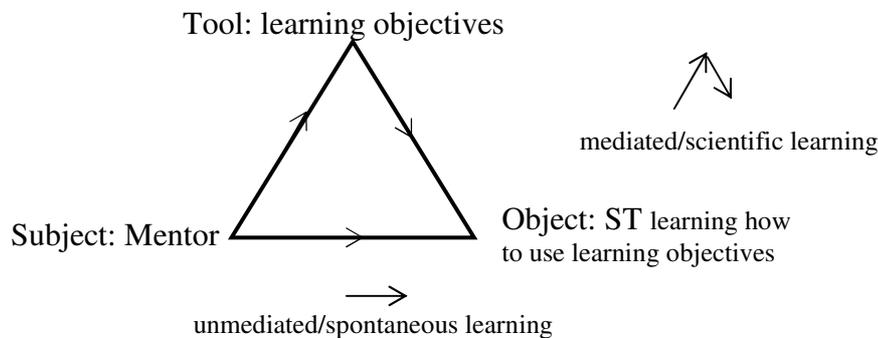


figure 1

Ways of Thinking about Professional Learning

Learning a professional practice, or developing expertise, is frequently seen as a developing capacity (a) to interpret aspects of the field of action in increasing complex ways and (b) to respond to those interpretations (Eraut, 1994; Sternberg and Horvath, 1995). A sociocultural approach to learning echoes this analysis. In sociocultural terms professional learning is evident in the capacity to interpret the 'object of our activities' (i.e. what it is we are focusing our energies and attention on) so that its complexity is increasingly revealed. For example, a ST might see a pupil as troublesome, but after conversations with a mentor or tutor might learn to interpret that behaviour as troubled and revise any responses (Edwards and Protheroe, 2004). In sociocultural analyses this process is called 'expanding the object' (Engeström, 1999), i.e. seeing more of the potential meanings in an event. STs work hard to avoid the unexpected while teaching (Desforges, 1995; Edwards, 1998), they avoid expanding the object, close down on complexity and limit their learning. STs need to be guided towards richer interpretations in the act of teaching with their learning being mediated by more expert teachers in the processes of interpretation and response in the classroom. (Edwards and Protheroe 2003)

A sociocultural approach also adds to understandings of expertise by seeing it as:

(an) ongoing collaborative and discursive construction of tasks, solutions, visions, breakdowns and innovations. (Engeström and Middleton, 1996, p. 4)

That is, expertise is not located within one individual but is distributed across systems in the forms of other people and the artefacts that they have produced (Hutchins, 1991; Pea, 1993), or as what Bruner has called the 'extended intelligence' of settings (Bruner, 1996). Expertise is a matter of informed interpretation of complex phenomena in professional practice and a form of resourcefulness which involves using the expertise of others in order to respond intelligently to those interpretations.

Ways of Thinking about Communities of Learning

The phrase community of practice comes from cognitive anthropology where it was used to explain how novice members of a community were inducted into the practices of more expert community members such as weaving material using traditional patterns, or learning how to cut and make up clothes (Lave and Wenger, 1991; Lave, 1997). Here the community could be easily defined and the practices were well-established and relatively unchanging. The metaphor for learning these practices in such communities is clearly participation.

There are socialisation versions of Initial Teacher Education (ITE) which accept this model of learning as STs are inducted into the communities of the schools in which they train. But we have to ask whether this process can provide sufficiently broad a professional training (Maynard, 2001) and whether it is suited to preparing STs to deal with uncertain futures and the complexity of teaching.

We are, however, able to be more precise about learning communities, if we draw on the work of Engeström who has undertaken developmental research with organisations which include schools, but also primary health care services, banks and multinational companies. His unit of analysis is what he calls the 'activity system' (Engeström, 1999), figure 2. His analysis asks us to look at that system as a learning zone.

The mentor-ST interaction is subject to the rules of the system; these rules come from both school and university. In terms of the division of labour this interaction is influenced by his mentor, his tutor, the class teacher and possibly learning assistants. The mentor belongs to two communities in this partnership between school and university; the community where the ST is a learner and the community where he is teacher and the community where the mentor teaches children and that where she develops new teachers.

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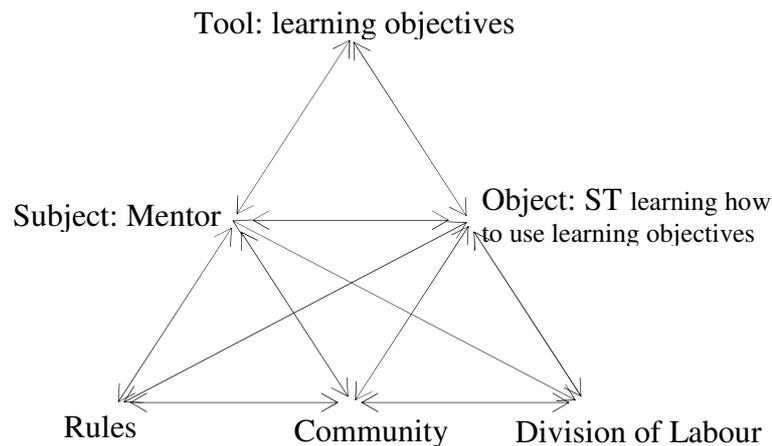


figure 2

The sociocultural approach suggests that we are both shaped by and shape the environments in which we participate (Cole, 1996). Knowledgeable teachers who can interpret complexity and act on those interpretations are therefore likely to be found in organisations which allow this to happen.

THE LESSON: AN ANALYSIS

A ST coming to the end of professional training taught the lesson described here. The lesson was observed by his school mentor (the head of department), and the university tutor. One of the authors observed the lesson and observed the debrief. The class was a challenging group of 14-15 year-olds and the topic was currency exchange.

The learning objectives were written on the board:

Learning objectives

- *to carry out conversions from pounds sterling to other currencies*
- *to convert from other currencies to pounds sterling*
- *to compare prices*

The main activity began with the first question: “Who is going on holiday this summer?” (practical wisdom, engage them in their ‘real world’). A couple of hands went up and they were asked, “Where to?” The response was a surprise, “Skeggie!” (Skegness; an English seaside resort). The question failed to engage; there was no need for the students to know about currency exchanges for their holidays.

We can pause here and ask questions about the activity system. To engage the pupils in the first of the learning objectives the ST decided to use something from the real world, advice that can be employed as rule for teaching, but in order to do that successfully you have to understand the community the children come from. In terms of division of labour, who else besides the mentor is responsible for helping the ST to understand what is real and relevant? Whose responsibility is it in this community to know about children’s cultural context?

The ST continued with his teaching, offering worked examples on the board. with the algorithm emphasised as:

$$£1 = \text{US\$ } 1.79$$

$$£2 = \text{US\$ } 3.58 \text{ “Multiply by 2 as we have 2 pounds”}$$

$$£5 = \text{US\$ } 8.95 \text{ “Multiply by 5”}$$

The activity was concluded with a repetition of the rubric, “To convert from pounds to other currencies we just multiply.” The students were then given a worksheet which contained the calculations to do, but gave no explanation or repetition of the worked examples. The routine was repeated for converting currencies to pounds using division.

The students had carried out the conversions as outlined in the learning objectives but as they had been told exactly what to do, we might say that they had done the conversions; literally by doing the calculations. The question for teacher education remains, had the students learned how to convert currency.

In his lesson a plenary was planned and used in the final five minutes. A problem was displayed using an overhead projector and it concerned finding the best price.

The price of a CD is quoted on two different internet sites is US\$15.95 and €17.91. The same CD costs £9.99 in HMV.

This was the only time the class was animated. The context was real this time, they understood the problem and we suspect they wanted to know how to do the mathematics. Two students immediately calculated the best deal; the first time they appeared to have done any work. Another student said that this choice was not necessarily correct as the postage and packaging rates had not been considered. The context here engaged the pupils so there was opportunity to expand the pupils’ learning, would the mentor talk about this in the debrief.

THE DEBRIEF: AN ANALYSIS

The lesson debrief began with the mentor focusing on the behaviour problems in the classroom. The ST and mentor discussed the difficulties of managing this difficult class. When asked, the ST’s major concern was that the students could have done more work. Individuals and their difficulties formed a major part of the debrief but there was no specific discussion of the content of the lesson. The ST was praised for his persistence and patience and was encouraged to find positive aspects of the lesson. There appeared to be a satisfaction that the lesson had gone as well as possible given the difficult students. In considering the activity system, the community depends on manageable behaviour if students are going to learn so it is an important aspect for the mentor to work on with his novice.

The tutor alone raised the issue of whether the learning objectives had been met. The ST seemed convinced that the objectives had been met, citing the exercises that the students had completed. The interpretation might be that the ST’s learner-knowledge

told him that these were the calculations that he did when finding exchange values and therefore this is what he had to teach. Challenged further as to whether the algorithms, ‘just multiply’, ‘just divide’ were sufficient for the students to be said to be carrying out conversions, both the ST and the mentor gave no response. In relation to the activity system our interpretation is that they closed down the complexities of the use and role of learning objectives and thus limited the learning for the ST. In relation to Askew’s (2004) analysis we would say that the ST had good subject matter knowledge. What appears to be the issue is that he sees the doing of the conversions as synonymous with learning how to do the conversions when the former is procedural knowledge and second demands an understanding of which procedures to use. The mentor did not see the unsuccessful use of learning objectives as an opportunity to work on expanding the ST’s understanding of how the learning objectives might inform his teaching choices. For both the mentor and the ST the use of learning objectives are unproblematic, because they are rules of the activity system, agreed by the community. They are not integral to learning but are an expected part of the performance of the lesson. Their use was not the focus of a pedagogic exchange between the ST and the mentor. Our experience is that without such a pedagogical exchange, it is unsurprising that the ST did not fully integrate the use of learning objectives into his planning for learning.

DISCUSSION

This particular example reflects many similar instances observed in our practice (70 observations each per year). From a sociocultural perspective, we would now argue that if learning objectives are to be useful in the classroom they need to be developed as ‘scientific concepts’ within the interactions of mentor and ST to develop the spontaneous learning gained by the ST. In an activity system, the existence of the tool (write learning objectives) in itself is not sufficient to influence learning, as the example reveals. From observation, the students did not attain the learning objectives and the ST was not asked to evaluate their role in the lesson.

Engeström (1999) suggests that we need to consider other aspects of the system, the rules, the community and the division of labour to explain the almost irrelevant use of tools such as learning objectives. In this example, the stating of learning objectives has the status of rules. The community, the mathematics department, claimed in their documentation that they serve a valuable purpose. This led to the ST stating them without linking them to the learning of the students. In relation to division of labour the ST was left to make sense of their use, without a supporting pedagogic exchange. This example, we believe, reveals that it is not the lack of subject knowledge that leads to learning objectives being ill-connected to learning (Askew, 2004), but that the rule of stating them is considered sufficient to ensure learning. Division of labour can be problematic when there is a sharing of responsibility for the ST’s learning in the school-based context. The ST can be torn between the demands of mentor and tutor. Tutor and mentor may focus on differing aspects.

The role of rules, community and division of labour can be seen in many instances. Ponte et al. (2005) highlighted how the role of the tutor has to change in the communication by e-mail and forum. The community of practice (Gomez and Rico, 2005) may not be a community and see working in groups as a rule rather than a learning tool, thus preventing any development of shared expertise. The division of labour is highlighted in the area when the groups come to wrong mathematics, what role does the tutor play then.

Returning to lesson objectives, the 'Framework for Teaching Mathematics' states that the main part of the lesson will be more effective if you "make clear to the class what they will learn" (DfEE, 2001, p 1/28) and that "better standards of mathematics will occur when ... lessons have clear objectives." (ibid, p 1/6). The status of the Framework (or National Strategy) in English schools has led to an uncritical use of learning objectives with an expectation that they will be used in every lesson and teachers have to account for them in their records. Arguably an over-emphasis on such forms of accountability can restrict the flexibility necessary for accessing the extended intelligence of an organisation, or expanding one's interpretations of events. We need to ask whether there is enough flexibility in the system to allow for learning, i.e. for new interpretations of the object of activity to play into community understandings.

For teacher education the sociocultural activity system offers a language to analyse the situation within which our STs find themselves. The current challenge for teacher education in England, where there is an overabundance of government initiatives in classroom action, is to work with new teachers on the critical analysis of the purpose of such expectations as the writing of lesson objectives if lesson objectives are to be tools not rules.

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THE KNOWLEDGE QUARTET: CONSIDERING CHLOE

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Abstract: *In this paper we describe a framework for the identification and discussion of prospective elementary school teachers' mathematics content knowledge as evidenced in their teaching. This framework - 'the knowledge quartet' - emerged from intensive scrutiny of 24 videotaped lessons. Application of the 'quartet' in lesson observation is illustrated with reference to a fragment of a lesson taught by one trainee teacher.*

Keywords: teacher education, teacher knowledge, classroom observation.

INTRODUCTION

At the previous CERME meeting, in Bellaria, Italy, we gave a paper (Rowland, Thwaites and Huckstep, 2004) in which we discussed the mathematics teaching practices of 12 prospective elementary teachers with reference to the subject matter knowledge (SMK) and pedagogical content knowledge (PCK) that seemed to underpin their pedagogical decisions and actions. Towards the end of the paper we hinted at a conceptualisation of teacher content knowledge in action which was, at that time, just emerging in our thinking. Assisted by the feedback that we received at CERME3, our thinking and understanding has moved on. We present here an account of where it stands at present, and how this research is being applied in our own initial teacher education programmes, and how this, in turn, raised additional questions for fundamental research. This exemplifies nicely the dialectical relationship between theory and practice, and the 'theoretical loop' which is the subject of Skott's (2005) paper in this volume. Specifically, we shall describe a research-based framework which facilitates the discussion of mathematics SMK and PCK between prospective teachers, their mentors and teacher educators. We begin with a brief résumé of our purposes and methods.

PURPOSE OF THE RESEARCH

In the UK, most trainee teachers follow a one-year, postgraduate course leading to a Postgraduate Certificate in Education (PGCE) in a university education department. Over half of the PGCE year is spent working in schools under the guidance of a school-based mentor. Placement lesson observation is normally followed by a review meeting between a school-based teacher-mentor and the student-teacher ('trainee', in the terminology of recent official UK documentation). On occasion, a university-based tutor will participate in the observation and the review. Research shows that such meetings typically focus heavily on organisational features of the lesson, with very little attention to mathematical aspects of mathematics lessons (Brown, McNamara, Jones and Hanley, 1999; Strong and Baron, 2004). The purpose of the research reported in this paper was to

develop an empirically-based conceptual framework for the discussion of the role of trainees' mathematics SMK and PCK, in the context of lessons taught on the school-based placements. Such a framework would need to capture a number of important ideas and factors about content knowledge within a small number of conceptual categories.

METHOD

This study took place in the context of a one-year PGCE course, in which 149 trainees followed a route focusing either on the 'lower primary' years (LP, ages 3-8) or the 'upper primary' (UP, ages 7-11). Six trainees from each of these groups were chosen for observation during their final school placement. Two mathematics lessons taught by each of these trainees were observed and videotaped, i.e. 24 lessons in total. We took a grounded theory approach to data analysis (Glaser and Strauss, 1967). In particular, we identified aspects of trainees' actions in the classroom that could be construed to be informed by their mathematics SMK or PCK. These were located in particular moments or episodes in the tapes. This inductive process generated a set of 18 codes. This was valuable from the research perspective, but presented us with a practical problem. We intended to offer our findings to colleagues for their use, as a framework for reviewing trainees' mathematics content knowledge from evidence gained from classroom observations of teaching. We anticipate, however, that 18 codes is too many to be useful for a one-off observation. Our resolution of this dilemma was to group them into four broad, super-ordinate categories, or 'units', which we term 'the knowledge quartet'.

FINDINGS

We have named the four units of the knowledge quartet as follows: foundation; transformation; connection; contingency. Each unit is composed of a small number of cognate subcategories. For example, the third of these, *connection*, is a synthesis of four of the original 18 codes, namely: *making connections*; *decisions about sequencing*; *anticipation of complexity*, and *recognition of conceptual appropriateness*. Our scrutiny of the data suggests that the quartet is comprehensive as a tool for thinking about the ways that subject knowledge comes into play in the classroom. However, it will become apparent that many moments or episodes within a lesson can be understood in terms of two or more of the four units; for example, a **contingent** response to a pupil's suggestion might helpfully **connect** with ideas considered earlier. Furthermore, it could be argued that the application of subject knowledge in the classroom *always* rests on **foundational** knowledge. Drawing on the extensive range of data from the 24 lessons, we offer here a brief conceptualisation of each unit of the knowledge quartet.

Foundation

The first member of the quartet is rooted in the foundation of the trainees' theoretical background and beliefs. It concerns trainees' knowledge, understanding and ready recourse to their learning *in the academy*, in preparation (intentionally or otherwise) for their role in the classroom. It differs from the other three units in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use. This distinction relates directly to Aristotle's account of 'potential' and 'actual' knowledge.

“A man is a scientist ... even when he is not engaged in theorising, provided that he is capable of theorising. In the case when he is, we say that he is a scientist in actuality.” (Lawson-Tancred, 1998, p. 267). Both empirical and theoretical considerations have led us to the view that the other three units flow from a foundational underpinning.

A key feature of this category is its *propositional* form (Shulman, 1986). It is what teachers learn in their ‘personal’ education and in their ‘training’ (pre-service in this instance). We take the view that the possession of such knowledge has the potential to inform pedagogical choices and strategies in a fundamental way. By ‘fundamental’ we have in mind a rational, reasoned approach to decision-making that rests on something other than imitation or habit. The key components of this theoretical background are: knowledge and understanding of mathematics *per se*; knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics; and espoused beliefs about mathematics, including beliefs about why and how it is learnt.

Transformation

The remaining three categories, unlike the first, refer to ways and contexts in which knowledge is brought to bear on the preparation and conduct of teaching. They focus on knowledge-in-action as *demonstrated* both in planning to teach and in the act of teaching itself. At the heart of the second member of the quartet, and acknowledged in the particular way that we name it, is Shulman’s observation that the knowledge base for teaching is distinguished by “...the capacity of a teacher to *transform* the content knowledge he or she possesses into forms that are pedagogically powerful” (1987, p. 15, emphasis added). This characterisation has been echoed in the writing of Ball (1988), for example, who distinguishes between knowing some mathematics ‘for yourself’ and knowing in order to be able to help someone else learn it. As Shulman indicates, the presentation of ideas to learners entails their re-presentation (our hyphen) in the form of analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). Our second category, unlike the first, picks out behaviour that is directed towards a pupil (or a group of pupils), and which follows from deliberation and judgement informed by foundation knowledge. This category, as well as the first, is informed by particular kinds of literature, such as the teachers’ handbooks of textbook series or in the articles and ‘resources’ pages of professional journals. Increasingly, in the UK, teachers look to the internet for bright ideas and even for ready-made lesson plans. The trainees’ choice and *use of examples* has emerged as a rich vein for reflection and critique. This includes the use of examples to assist concept formation, to demonstrate procedures, and the selection of exercise examples for student activity.

Connection

The next category binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content – the learning, perhaps, of a concept or procedure. It concerns the *coherence* of the planning or teaching displayed across an episode, lesson or series of lessons. Mathematics is notable for its coherence as a body of

knowledge and as a field of enquiry, and the cement that holds it together is reason. The pursuit of coherence and mathematical connections in mathematics pedagogy has been stimulated recently by the work of Askew, Brown, Rhodes, Wiliam and Johnson (1997); of six case study teachers found to be highly effective, all but one gave evidence of a 'connectionist' orientation. The association between teaching effectiveness and a set of articulated beliefs of this kind lends a different perspective to the work of Ball (1990) who also strenuously argued for the importance of connected knowledge for teaching.

In addition to the integrity of mathematical content in the mind of the teacher and his/her management of mathematical discourse in the classroom, our conception of coherence includes the *sequencing* of topics of instruction within and between lessons, including the ordering of tasks and exercises. To a significant extent, these reflect deliberations and choices entailing not only knowledge of structural connections within mathematics itself, but also awareness of the relative cognitive demands of different topics and tasks.

Contingency

Our final category concerns the teacher's response to classroom events that were not anticipated in the planning. In some cases it is difficult to see how they could have been planned for, although that is a matter for debate. In commonplace language this dimension of the quartet is about the ability to 'think on one's feet': it is about *contingent action*. The two constituent components of this category that arise from the data are the readiness to *respond to children's ideas* and a consequent preparedness, when appropriate, to *deviate from an agenda* set out when the lesson was prepared. Shulman (1987) proposes that most teaching begins from some form of 'text' - a textbook, a syllabus, ultimately a sequence of planned, intended actions to be carried out by the teacher and/or the students within a lesson or unit of some kind. Whilst the stimulus - the teacher's intended actions - can be planned, the students' responses can not. Ainley and Luntley (2005) suggest that experienced teachers' ability to respond effectively to unpredicted events during lessons draws on what they call 'attention-based knowledge', which is somehow different from SMK and PCK.

Brown and Wragg (1993) suggest that 'responding' moves are the lynch pins of a lesson, important in the sequencing and structuring of a lesson, and observe that such interventions are some of the most difficult tactics for newly qualified teachers to master. The quality of such responses is undoubtedly determined, at least in part, by the knowledge resource available to the teacher.

CHLOE'S LESSON

We now proceed to show how this theoretical construct, the knowledge quartet, might be applied, by detailed reference to a 14-minute portion of one of the 24 videotaped lessons. The trainee in question, Chloe, was teaching a Year 1/2 (pupil age 5-7) class a particular strategy for mental subtraction. By focusing on this vignette we aim to maximise the possibility of the reader's achieving some familiarity with the scenario, with Chloe and a few of the children in her class. What is lost, of course, is any sense of how the quartet might inform reflection on the rest of her lesson. On the other hand, the passage we have

selected would be, in itself, a valuable focus for some useful reflection in the post-lesson mentoring discussion.

Conforming to the English *National Numeracy Strategy* (NNS) guidance (DfEE, 1999), Chloe segments the lesson into three distinctive and readily-identifiable phases: the *mental and oral starter*, the *main activity* and the concluding *plenary*. The learning objective stated in Chloe's lesson plan is: "Children should be able to subtract 9, 11, 19 and 21 using the appropriate strategies". The lesson begins with a three-minute *mental and oral starter*, in which Chloe asks a number of questions such as 'How many must I add to 17 to make 20?', designed to test recall of complements of 10 and 20. There follows a 14-minute *introduction* to the *main activity*. Chloe reminds the class that in their previous lesson they added 9, 11, 19 and 21 to various 1-digit and 2-digit whole numbers. Chloe demonstrates how to subtract these same numbers by subtracting 10 or 20 first, then adding or subtracting 1. She models the procedure, moving a counter vertically and horizontally on a large 1-100 square. At the end of the demonstration, Chloe lists an example of each of the four subtractions on a whiteboard. The class then proceeds to 23 minutes' seatwork on differentiated worksheet exercises that Chloe has prepared. The 'more able' children subtract 19 and 21, the others subtract 9 and 11. Finally, she calls them together for a four-minute *plenary*, in which they consider $30 - 19$ and $43 - 21$ together.

Chloe's Lesson and the Knowledge Quartet

We now home in on the introduction to the main activity, to see how it might be perceived through the lens of 'the knowledge quartet'. This is typical of the way that the quartet can be used to identify for discussion various matters that arise from the lesson observation, and to structure reflection on the lesson. Some possibilities for discussion with the trainee, and for subsequent reflection, are flagged below thus: **Discussion point**. We emphasise that the process of *selection* in the commentary which follows has been extreme.

Foundation: Chloe's lesson plan refers to "appropriate strategies" for subtracting four near-multiples of 10, without recording what strategies she has in mind. It becomes clear that she will emphasise mental, sequential strategies, perhaps with some use of informal jottings (DfEE, 1999, p. 2/4). This is very much in keeping with the *National Numeracy Strategy*, which, following the Dutch RME (Realistic Mathematics Education) approach, emphasises mental calculation methods in the early grades. Sequential (or cumulative) strategies for two-digit addition and subtraction begin with one number (for subtraction, the minuend) and typically move up or down the sequence of integers in tens or ones. Split-tens methods, by contrast, partition both numbers into tens and units and operate on the two parts separately, before re-combining (e.g. Anghileri, 2000, pp. 62-65). The objective of the previous lesson (on adding near-tens) and the current one is taken directly from the NNS *Framework* (DfEE, 1999) teaching programme for Year 2:

Add/subtract 9 or 11: add/subtract 10 and adjust by 1. Begin to add/subtract 19 and 21: add/subtract 20 and adjust by 1. (p. 3/10)

These objectives are clarified by examples later in the *Framework*; such as

$58+21=79$ because it is the same as $58+20+1$; $70-11=59$ because it is the same as $70-10-1$

$24-9=15$ because it is the same as $24-10+1$; $35+19=54$ because it is the same as $35+20-1$ (p. 4/35)

The superficial similarity in these examples; captured in the NNS objective immediately above, is, we would suggest, deceptive. The differences between them can be articulated in terms of what Marton and Booth (1997) call ‘dimensions of variation’. The dimensions in this case bring with them different kinds and levels of complexity, as follows.

Dimension 1: Addition or subtraction. In general terms, it might be thought that subtraction is the more demanding. Indeed, the first lesson of the two had dealt exclusively with addition, the second with subtraction.

Dimension 2: Near multiples of 10 or 20. Again, it seems reasonable to anticipate that adding/subtracting 20 is the more demanding. Indeed, Chloe has explicitly planned for the lower-attaining groups of pupils to work exclusively with 9 and 11.

Dimension 3: One more or one less than 10/20. Addition and subtraction of 11/21 entail a sequence of actions in the *same* direction i.e. aggregation or reduction; whereas 9/19 require a *change* of direction for the final unit i.e. compensation. Research confirms what might be expected, that the latter is less spontaneous and more demanding (e.g. Heirdsfield, 2001). Indeed, the compensation strategy for adding/subtracting 9 is, in lay terms, a ‘trick’.

Discussion point: what considerations determined Chloe’s choice of worksheet problems for the two ‘ability’ groups in the class?

Transformation. We pick out two factors for consideration relating to this dimension of the quartet (as usual, bearing in mind that they are underpinned by foundational knowledge). First, Chloe’s use of the 100 square as a model or representation of the sequence of two-digit positive integers. The 100 square is useful for representing ordinal aspects of the sequence, though with some discontinuities at the ‘ends’ of the rows, and particularly for representing the place-value aspects. Chloe makes full use of the 100 square in her exposition, but is frequently dismissive of children’s use of the spatial language that it invites. For example, at one point she places the counter on 70:

Chloe: Right, there’s 70. [...] From 70 I want to take away nine. What will I do?
Rebecca?

Rachel: Go up one.

Chloe: No, don’t tell me what I’m gonna go up or move, tell me what I actually do.

Rachel: Take away one.

Chloe: Take away one to take away nine? No. Remember when we added nine we added ten first of all, so what do you think we might take away here?
Sam.

Simon: Ten.

This would seem to relate to the format of the NNS examples (above), which she follows in four ‘model’ solutions that she writes for reference on the board, e.g.

$$70 - 9 = ?, 70 - 10 + 1 = 61$$

Somewhat surprisingly, the children are forbidden to use 100 squares when they do the worksheet exercises. Chloe refuses a request from one child for a “number square”, saying, “I want you to work them out all by yourselves”. In fact, there is nothing in Chloe’s lesson plan to indicate that she had intended to use the 100 square in her demonstration.

Discussion point: What led Chloe to use the 100 square? What are its potential affordances - and constraints - for calculation relative to the symbolic recording in the NNS examples? Had she considered using an empty number line (e.g. Rousham, 2003) as an alternative way of representing the numbers and their difference, of clarifying when compensation is necessary, and why?

The other aspect of transformation that we select here concerns Chloe’s choice of examples. Space considerations restrict us to mentioning just one, in fact the first chosen to demonstrate subtraction, following the initial review of addition. Chloe chooses to subtract 19 from 70. We have already argued that subtracting 11 and 21 would be a more straightforward starting point. Moreover, 70 is on the extreme right boundary of the 1 to 100 square. After moving up two squares to 50, there is no ‘right one’ square: it is then necessary to move down and to the extreme left of the next row, so the neat ‘knights move’ is obscured, and the procedure unnecessarily complicated. We note that one of the NNS *Framework* examples (above) is $70 - 11$, and that all four of Chloe’s whiteboard template examples were of the form $70 - n$.

Discussion point: Was Chloe aware in-the-moment of the complication mentioned above, or did she anticipate it in her planning? Did the symbolic form in her written plan ($70 - 20 + 1$) perhaps obscure the consequences of her using the 100 square for this calculation?

Connection. Chloe makes explicit links with the previous lesson on adding near-multiples of 10, and reviews the relevant strategies at the start of this one. Her oral and mental starter, on complements to 10 and 20, essentially focuses on the concept of subtraction as comparison, whereas the strategy taught in the main activity is on change-separate, or ‘take away’, subtraction (Carpenter and Moser, 1983). Procedures associated with the two concepts tend to be based on strategies for counting on and counting back respectively (*ibid.*). Arguably Chloe could have encouraged some flexibility in the choice of such procedures, whereas she chose to prescribe exclusively forms of counting back in the main activity. The effect of her approach to differentiation for the different groups was to emphasise the similarity between 9 and 11 (needing an initial

subtract-10) and between 19 and 21 (subtract-20), when the pairing of 11 and 21 (consistent reduction) and 9 and 19 (needing compensation) was an alternative form of connection.

Given her use of the 100 square to demonstrate the strategies, there was scope for some discussion of the links between vertical and horizontal spatial movements on the board and the tens-ones structure of the numbers under consideration. As we have remarked, she actively discouraged children's reference to the spatial analogue. It seemed that her attention was on conformity at the expense of flexibility and meaning-making.

Discussion point: discussion could usefully focus on the two subtraction concepts, how they relate to the first two phases of the lesson, and whether comparison strategies might offer useful alternatives to 'take-away with compensation', in the case of subtracting 9/19.

Contingency. A key component of our conceptualisation of this dimension of the quartet relates to how the teacher responds to unexpected or deviant ideas and suggestions from children in the lesson. There are no compelling distractions from Chloe's planned agenda for the lesson in this episode, although the child's question about using the number squares for the exercises might be a case in point. Various children's use of up/down language on the 100 square, to which we have already referred, might have been usefully explored rather than dismissed. A similar opportunity presented itself when, in the review of adding 9 at the beginning of the lesson, Chloe invites one of the pupils to demonstrate:

Chloe: Show the class how you add ten and take away one on a number square.
What's the easy way to add ten on a number square? Cameron.

Cameron: Go diagonally.

Chloe: Not diagonally. To add ten you just go...

Cameron: Down.

No further reference is made to Cameron's diagonal proposal, although his elegant use of vocabulary alone is surely worth a moment's pause. It is true that his initial suggestion is not, strictly, a correct answer to her "add ten" question. It does, however, offer a nice spatial way of thinking about adding 9 - and adding 11 too - and suggests that Chloe's mentor may have stressed it in the previous lesson. Indeed, the fact that adding 9 corresponds to a diagonal south-west move might usefully connect to the insight that subtracting 9 would necessitate a north-east move, and the consequent need to *add* one after subtracting 10. It would seem that Chloe is too set on her own course to explore the possibilities offered by remarks such as Cameron's.

Discussion point: Did Chloe recall Cameron's suggestion? If so, how did she feel about it at the time, and how might she have responded differently?

It is important to add that the second of these questions is sincerely asked: there are often very good reasons for teachers sticking to their chosen path. The purpose of the question is to raise awareness of the fact that an opportunity was presented, and that a different

choice could have been made. We also reiterate that a single event or episode can frequently be considered from the perspective of two or more dimensions of the quartet, as demonstrated in our commentary.

FINAL COMMENTS AND CAVEATS

In this paper, we have introduced ‘the knowledge quartet’ and shown its relevance and usefulness in our analysis of part of Chloe’s lesson with a Year 1/2 class. We have a manageable framework within which to discuss actual, observed teaching sessions with trainees and their mentors. Within the last year, the four dimensions of the knowledge quartet have been used as a framework for lesson observation and reflection in the context of our own university’s pre-service elementary and middle school teacher education programmes. Initial indications are that this development has been well received by mentors, who appreciate the specific focus on *mathematics* content and pedagogy. They observe that it compares favourably with guidance on mathematics lesson observation from the NNS itself, which focuses on more generic issues (DfEE, 2000, p.11).

It is all too easy for an observer to criticise a novice teacher for what they omitted or committed in the high-stakes environment of a school placement, and we would emphasise that the quartet is intended as a tool to *support* teacher development, with a sharp and structured focus on the impact of their SMK and PCK on teaching. Indications of how this might work are explicit in our analysis of Chloe’s lesson. We have emphasised that our analysis has been *selective*: we raised for attention some issues, but there were others which, not least out of space considerations, we chose not to mention. The same would be likely to be true of the review meeting - in that case due to *time* constraints, but also to avoid overloading the trainee with action points. Such a meeting might well focus on a lesson fragment, and on only one or two dimensions of the knowledge quartet for similar reasons.

Any tendency to descend into deficit discourse is also tempered by consideration of the wider context of the student teacher’s experience in school. In the novice teacher we see the very beginnings of a process of reconciliation of pre-existing beliefs, new ‘theoretical’ knowledge, ‘practical’ advice received from various quarters, in the context of highly-pressured, high-stakes school-based placements. There is also good evidence (e.g. Hollingsworth, 1988; Brown, Mcnamara, Jones and Hanley, 1999) that trainees’ concern for pupil learning is often eclipsed by their anxieties about timing, class management and pupil behaviour. Skott (2001) gives instances of three novice teachers’ actions in the classroom that were at odds with their strongly reformist priorities. Sometimes these were due to mathematical insecurity, but often they reflected pedagogical and social agendas (such as building self-confidence) that transcended pupil learning. We recognise that our “purely mathematical perspective” (*ibid.*, p. 193) has its limitations in coming to understand why teachers do what they do.

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THE PORTFOLIO IN TEACHER EDUCATION

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Abstract: *This paper focuses on the production of a portfolio in the subject of Didactics of Mathematics, a component of the study plan of Mathematics preservice teacher education. We present some evaluation results, showing how the students view their experiences, which kind of learning they developed and the main role of the reflections. Finally, we conclude with a discussion about the potential of the portfolio in preservice teacher education, some of the difficulties students encounter and the required conditions to throughout the process.*

Keywords: teacher education; mathematics education; reflection; assessment; monitoring process; portfolios.

INTRODUCTION

In Portugal, initial teacher education follows several models, depending on the teaching level and the educational institution. The School of Sciences of the Lisbon University trains Mathematics teachers for the 3rd cycle of basic education (students from 12 to 14 years old) and for secondary education (students from 15 to 17 years old). During the first three years, the students only have Mathematics, under the responsibility of the Mathematics Department; in the fourth year they have Educational subjects and Didactics of Mathematics, taught by the Department of Education; and in the fifth year they teach Mathematics at a school, with the supervision of a teacher from the school and two from the university, one of mathematics and one of didactics.

The subject Didactics of Mathematics is annual, with 5h of class per week. General themes of Mathematics teaching and learning are discussed, such as the Mathematics curriculum, the Mathematics class, current problems in education and the Mathematics teacher, and specific aspects of Mathematics teaching themes are worked on, such as Geometry, Algebra and Functions, Numbers and Statistics and Probabilities.

In 2003-04 school year, for the first time the elaboration of a portfolio was introduced as one of the assessment tools. We intended to create opportunities for preservice teachers to reflect about the work they were developing and to provide them with the experience of working on with this tool.

THE PORTFOLIO AS AN ASSESSMENT TOOL

Over the last decade the portfolio has been used as an alternative assessment tool of students at different levels of schooling and especially in initial and in-service teacher education (Porter et al., 2001; Wade & Yarbrough, 1996). But resorting to the

portfolio means more than using a new assessment tool. Above all, it is a theoretical action (Shulman, 1999), for it implies a set of presuppositions regarding learning and evaluation. Learning is seen as an action developed by the subject through meaningful, relevant experiences, whose interaction with others constitutes a favourable context. Assessment, as a monitoring process of learning, should contribute to pertinent, contextualised work, that calls for reflexive thinking, that allows and facilitates meta-cognition (Hadgi, 1997), teamwork and engagement, responsibility and affectivity (Forgette-Giroux & Simon, 1997).

The portfolio can have two different purposes. During its construction process it may contribute importantly to learning, through self-evaluation, external feedback and reflection about what was learnt and how and the identification of strengths and weaknesses (Tillema, 1998). When it is finished, it permits to access the student's evolution over a vast period of time, such as a school year (Clarke, 1996).

By portfolio we mean a diversified and representative sample of work produced by the student over a vast period of time. It is up to the student to choose each work that is to be included in his portfolio. Each one must be accompanied by a personal reflection that explains the meaning that the work had for the student. A final reflection must also be included at the end, about the work carried out and its contribution to the student's learning. Therefore, the portfolio is characterised by the set, selection and organisation carried out by the student and shows his reflections and learning (Wade & Yarbrough, 1996).

So the portfolio is a means to develop the student's ability to reflect about what he did and how he did it and to give him more autonomy for making decisions, both in choosing the materials that constitute the portfolio and in organising it, thus allowing the student a more active role in his own assessment (Clarke, 1996). But the portfolio also brings advantages to the teacher. The accompanied construction process of this tool narrows student-teacher communication, allowing the teacher to get to know the student in greater depth.

However, certain conditions are essential and some risks are to be avoided. Both teacher and students have to assume that the serious construction of a portfolio takes a long time (Shulman, 1999) and goes on for a vast period. A portfolio cannot be produced in one afternoon, neither can include just one or two items (Wade & Yarbrough, 1996). They must also be aware of the risk of (i) trivialising it, by including items that are not worth reflecting about; (ii) turning it into a simple exhibition of the best we can do, while devaluing a context that is favourable for reflection; and (iii) twisting its nature, establishing very objective criteria in order to establish comparisons among students (Shulman, 1999).

METHODS

Context for the study

My Didactics of Mathematics class in the 2003-04 school year had 9 boys and 19 girls. On the first day of school, when the subject's program was presented, the

students were informed that they would have to elaborate a portfolio. As this was for all of them a first experience, I was aware of the difficulty in fully understanding immediately what I was asking them to do (Wade & Yarbrough, 1996). Therefore, a small document was distributed, explaining what was intended. It stated what the portfolio should include (index, introduction, 6 to 8 tasks, each accompanied by a reflection and a final reflection), the criteria for choosing the tasks to be included in the portfolio (being representative of the diversity of the nature of the work developed and of the themes handled in this subject) and the evaluation criteria (content 80%; presentation and organisation 20%). Each task had to indicate the date it was included in the portfolio. A task included in the portfolio could later be replaced by another, and this replacement had to be adequately justified.

On that same day, we negotiated which was to be the first class devoted to supporting the students in elaborating their portfolios. Actually, instead of one class two were taken up and this task was completed with other moments of support outside classes. As the reflection was what students found most difficult to do, each one read a reflection that was already written and the other elements of the groups made comment on it, after which I concluded with my comments, many times in the form of a question. While I supported one group, the remaining students, also in groups, shared their difficulties. A co-evaluation began to develop. At the end of the first semester, I took all the portfolios home and commented them one by one, highlighting what they had already achieved and what still needed to be improved. During the second semester there were no classes dedicated to the portfolios, but once in a while the students bring their questions to me.

Procedures

The study used three data collection methods: the sessions of monitoring the process that was audio-taped; document analyses (the portfolios) and a final questionnaire applied to all the students. This questionnaire has only open-end questions¹. Applied once the school year had ended, this questionnaire was anonymous so as to give the students the chance to express their opinions without feeling any sort of constraint. 23 (82%) students answered the questionnaire.

The data were submitted to content analysis concerning three fields: the perspectives that the student teachers had faced, the kind of learning they got through the construction of this tool and its contributions for the development of their abilities to reflect. Through document analysis, in particular, the portfolio of each student teacher and their answers got by the questionnaire, the different contents have been coded and grouped by their meanings. In this way, the categories of the analysis were constructed as the data analysis was developed.

¹ The questionnaire had 7 questions, but in this text only two questions will be used.

Question2: What kind of learning do you think you achieved with the elaboration of the portfolio?

Question6: What are the main differences you identify between this assessment process and others that you knew?

RESULTS

The portfolio

As expected, the students had never come into contact with this tool, so at the beginning they did not know its meaning and even wondered why it emerged in this subject: “I couldn’t understand why a future Mathematics teacher had to do something like that now” (T)². Some of them decided to find out more about the portfolio, by reading or on the Internet: “I immediately went to look it up, I consulted books and did research on the Internet” (L).

One year later, the students stress the main differences they identify in the portfolio when compared to other assessment tools they know, particularly written tests or exams, the most frequently used forms throughout their students’ experiences. One of the differences they point out concerns the object of assessment that, in their opinion, did not fall upon specific knowledge, but instead had a wider scope, emphasising different types of higher-level capacities, namely meta-cognition:

The other assessment processes assess the specific knowledge we acquire directly. But the portfolio seems to assess our thinking about acquired knowledge. (4)

The fact that it is a construction process that develops over time, “without stress”, favours the monitoring character of assessment, especially self-evaluation:

It’s a work that’s never finished and that can always be improved. (14)

The fact that halfway through the portfolio elaboration we were given an opinion about our work up until then guided our work performance in a positive way. (17)

Each improved task made me go over the same content, self-evaluate the work that was being done. (F)

The moments of interaction that occur between teacher and students as well as the fact that the students had the opportunity to write what they think about several issues, that are particularly important to them, seem to contribute to “a greater student/teacher proximity, giving the latter a better chance to identify difficulties and help the students” (16). Particularly for the shyer students, the portfolio may be an opportunity to show who they really are:

The portfolio has a crucial utility for students like me. I’m one of those who easily go unobserved in classes. I don’t like to participate, I probably even show discouragement, but I simply prefer to hear the explanations and reflect about them internally. This way, this work shows the teacher the attention, interest and dedication that I had. (F)

² When a letter is used, we are referring to a portfolio, a number refers to an answer in the questionnaire.

But the biggest difference students found is related to the high level of autonomy they are given in the process, associated with a greater accountability that is required: “The portfolio ends up being a big responsibility for me. With it I truly have the notion of being in the centre of the teaching-learning process” (E).

The students’ active role in the assessment process is heavily related to the personal character the portfolio assumes to them. As one student explains, “this document isn’t just an assessment work, it’s also the representation of my thoughts, my ideas” (F). So, being a document in which students reveal a lot of themselves, much of what is often in their intimacy, it is understandable that they grow attached to the portfolio that they see as an extension of themselves:

I have never had a diary but I think this experience was quite similar, although I don’t describe my days here I describe my thoughts, and so it becomes much more intimate and personal. The separation’s going to be hard and I really think I’m going to miss it. (E)

The learning process

The students feel that they have learned a lot while they developed the portfolio. More specifically, the need to be organised was one of the aspects that emerge: “Choosing the tasks for the portfolio allowed me to develop my organisational capacity” (8). Actually, from my observations of the classes and compared to previous years, I noticed that these students showed more concern in recording the discussions that took place in class and in keeping all the documents they did as they knew they might need them in the future, for their portfolio.

Considering that in previous years they had little or no work in terms of written documents, developing arguments through writing was another thing the students refer they learned: “I’ve always found it very hard to express myself in writing. With this work, I think I developed this competency quite a lot and I actually improved quite a lot” (AF). In fact, on the last day of school, when we were making a balance about the portfolios, the majority of the students said they had turned to other people’s help to improve their writing in the documents they elaborated. This attitude shows, on the one hand, that they were set on doing good quality work and, on the other, that they acknowledged their difficulties in this dimension.

Naturally themes related to the teacher’s practice were referred to, as well as bibliographical research, but I think this kind of learning has not exclusively to do with the portfolio. However, knowledge of an alternative assessment tool is another thing they learned that is clearly related to this specific work: “I got to know a new assessment method and the difficulties students may have carrying it out” (14). According to the procedures that were developed, the fundamental role the teacher gave the students was highlighted: “I learned that the comments a teacher makes on a student’s work have a lot to be said, you can’t say too much, but you can’t say too little” (A), as well as the importance of co-evaluation: “once more I can follow the teacher’s example and ask my students to read their group colleagues’ portfolios and

comment on them. That would be a great way to value discussion and a critical spirit” (E).

The fact that they had to choose tasks led many students to improve their first problem solution, carried out individually or in group. So, “thinking more before doing anything, and acknowledging mistakes” (16) contributed to learning in a more meaningful and permanent manner over time: “the activities I chose became more important and what I learned with them became really solid and expressive” (T).

But undoubtedly the fact that they had to reflect about what they did, how they did it and the issues that emerged from these experiences seems to have marked the students the most in the whole experience. Above all, it allowed them to get to know themselves better: “I think the most important thing I learned was getting to know myself better, not just as a future teacher, but also as a person” (4).

Reflection

The greatest difficulty the students faced when they did their portfolio was to have to develop a reflection to accompany each task they chose. Below is a good example of this difficulty, reported by a student:

[when I was told I had to do a portfolio] First I was scared, then curious, and then scared again (...) I avoided writing the first reflection for weeks, although after each class, on the bus on the way home, I imagined what I would write about what I’d just seen, but I never put it on paper. Until we got to the last day (...) it was very hard to write the first sentence. I had lots of ideas, but I didn’t know where to start. (...) When I finished and reread what I had written I couldn’t help smiling. At that moment I realised what the teacher meant when she talked to us about the importance of elaborating a portfolio. (T)

In the first meetings of joint work, most of the students presented rather impersonal reflections, mostly descriptive, with a low level of inquire. Most students’ evolution in this respect was not immediate, that is, it did not happen between the first moment of work and the end of the first semester. It was only after a second feedback, already in the second semester, that more generalised improvement was visible.

The open nature of the reflections may explain why some students referred that this task is always open to improvement: “the truth is this work is eternally imperfect” (E). As if this weren’t enough per se to make this work demanding, the possibility of replacing one task with another was grabbed by several students, not because they considered that what they had done was poor, but because something else had arisen that was more important to their learning:

When I decided to replace a reflection it wasn’t because I felt that the one I’d done was not worth anything, or that it was bad, but because related to that theme I’d reflected about, I found another one that seemed much more important to me and whose reflection could teach me more. (E)

While the reflections were a great challenge for the students, it is also true that having to reflect led them to develop this capacity and create a new stance, “I learned to create a habit of reflecting about situations I consider to be important” (11); “nowadays I am used to reflecting about the work I do and self-evaluating myself” (AF). It was also thanks to reflecting that some students questioned their concepts of teaching and learning Mathematics, heavily marked by their personal experience as students:

The reflections and even the analysis itself of the chosen situations were what made me change certain ideas I'd already formed about teaching Maths (A)

This is going to be my greatest difficulty because only throughout this year I had the opportunity to become aware of these changes and I know I'm going to have to fight hard not to yield to the temptation of going back to lessons of oral exposition/problem solving. (AF)

CONCLUSION

We may say that the production of a portfolio in the subject of Didactics of Mathematics was a successful experience, if we take into account what students say, and document in the portfolio, that they learned. More specifically, they developed their argumentation, writing ability, organisation, research, autonomy and responsibility in the learning process.

The monitoring role of this tool is probably its largest potentiality. The fact that the products of the chosen tasks and the first versions of reflections can be improved, based on the teacher's comments, creates most certainly new moments of learning (Tillema, 1998). The strong reflective component that was present throughout the whole process and moments of teacher/student interaction are the preferential means that allow students to develop their self-evaluative capacity (Hadgi, 1997; Jorro, 2000).

The development of an ongoing reflection based on specific tasks (Schön, 1983) allowed students to get to know themselves better as persons; to become aware of their one believes regarding teaching and learning and to question them in face of important issues of the teaching practice (Christiansen & Walter, 1986). Therefore, the portfolio constituted a favourable means for developing a reflective stance, a requirement currently considered to be essential to teachers (Mezirow, 1991).

What concerns Mathematics education, the student's teacher contacted and developed an alternative assessment instruments that focus in high level capacities instead on specific knowledge, valorising one of the trends of what is consider, in our days, knowing mathematics (NCTM, 2000).

But certain difficulties arise in developing a portfolio. The students need to engage in it seriously. It is a demanding task where they have to expose themselves. In order to do so, they must acknowledge its importance and the teacher/student relationship has

to be one of trust. The increase in work for the student and for the teacher is enormous – many hours, days, weeks! Studying the day before an assessment moment is not enough. It is ongoing work. Teachers must devote classes to this work, create different moments in teacher/student interaction, accompany and support their students. In short, it requires a new culture of evaluation, in which learning is the intended objective. How is it possible to prepare students and teachers for this culture of evaluation? How can they accept to spend so much time on this? How can we break away from such a strong-rooted, albeit currently questioned concept of assessment? These are questions that must be addressed in the future, so that the portfolio may become not the exception, but a more generalised practice, justified by its potentialities. These potentialities also included an important and even indispensable data for the teacher to get a deep understanding of the students' point of view of his or her one role. This dimension will be discussed in another paper.

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THE TEACHER'S ACTION, THE RESEARCHER'S CONCEPTION IN MATHEMATICS

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Abstract: *Our intention is to contrast the epistemological positions of teachers and researchers, by the means of their respective actions in a research process. Based on a threefold descriptive model of the teacher's action, our analyses examine the nature of the teaching techniques enacted about a given mathematical situation, the "Race to 20" (pupils aged 9-11 years) and the discourses of two teachers about this lesson. Our findings indicate that the teachers are primarily concerned with the educational coherence of the teaching process, using non specifically knowledge-related teaching techniques that researchers explain by some generic overdeterminations, from the didactical point of view. This gap has to be taken into account in further collaborative research, in order to make teacher's developing specific teaching techniques to foster the mathematical sense-making of the students.*

Keywords: Mathematics teaching, Race to 20, Teacher's action, Cognitive values, Mathematical knowledge-related teaching techniques, Epistemological gap.

This paper investigates the epistemological gap existing between two experienced teachers and a team of researchers, both involved in a particular research project. This gap is to be considered through the teacher's practices and discourses on one hand, and through the researchers' expectations and interpretations on the other hand. This study comes out as a side question in the main stream of our work on the teacher's action in the "Race to 20" mathematical situation.

A TEACHER'S ACTION MODEL AS THE MAIN FRAMEWORK

In order to understand the teacher's action while he carries out mathematics lessons, we designed a threefold model based upon didactical categories (Brousseau 1997 ; Chevallard, 1992). The first level of this theoretical frame comprises a set of *micro* teaching techniques, that were spotted several times among the interactions patterns in a given mathematical situation (Sensevy, Mercier, Schubauer-Leoni 2000). These techniques enable the researcher to describe in depth and precisely how a given piece of knowledge is handled by both the teacher and the students. A second level is also

defined to gather the *macro* shifts that can be observed in the didactical contract¹, and therefore it sums up the main teacher's intentions about the knowledge. Then, a third level is to take into account the teachers' general practices and educational beliefs through actions observed during the lesson, and/or comments we may get from the teachers during interviews. Moving up the levels in the model shows a "gradient" in the teaching techniques: at the bottom, are the most knowledge-linked techniques and at the top, are the less specific techniques. Actually, the two first levels are the core of our theoretical point of view, as we defend that the knowledge specificities and the corresponding teaching situations in mathematics are very likely to constrain the didactical interactions. In contrast to other pedagogical models designed to indicate what the teacher should do to achieve a "good" teaching practice, our model is built up empirically from classroom observations of teaching techniques, taking into account the didactical constraints of a given situation. As a descriptive tool at this stage, it is exclusively used for research analyses. Our comparative research project in didactics should contribute to sort out the generic or specific nature of the teaching techniques we identify.

In order to foster the validity of our empirical model, we conceived and carried out a research setup (Sensevy, Mercier, Schubauer-Leoni, Ligozat & Perrot, 2005) in which the didactic situation "Race to 20"² is still an experimental paradigm for the studying of teacher's work. It involved two teachers at the fifth grade of French primary school. The teachers were first trained to the mathematical issues of the "Race to 20" game and secondly, they were asked to carry out the "Race to 20" situation as a lesson with their respective classes. The teachers were free to plan the lessons as they wished.. A third phase consisted in the teachers' self-analyzing their first lesson³, based on a video recording of the lesson. A fourth phase consisted in the teacher's cross-analyzing their first lesson: T2 analyzed T1's lesson on the video recording (in presence of T1) and reciprocally. In this setup, the teachers are not part of the research team. They know each others quite well due to partnerships developed in other professional circumstances⁴, and we can say that interviews were carried out in a confident atmosphere. The researcher's work started afterwards, to carry out clinical analyses of the teaching techniques related to the "Race to 20" situation, encountered in the

¹ This concept was developed by Brousseau (1997) to describe the reciprocal expectations of the teachers towards the students and *vice versa*, about the meaning of a mathematical situation in which a knowledge is the stake.

² This fundamental situation in Brousseau's work (1997) is based upon a game which opposes two players. The first player says a natural number X_1 that is less than 3 (1, for example). The second player says a natural number Y_1 obtained by adding 1 or 2 to X_1 (for example, he says 3, a number obtained by adding 2 to 1). The first player then says a natural number X_2 , obtained by adding 1 or 2 to Y_1 (for example, he adds 1 and says 4), etc. The player who is the first to say 20 is the winner. There are numbers that it are sufficient to say in order to win: 2, 5, 8, 11, 14, 17, 20.

³ T1 and T2 gave more than one lesson on this subject.

⁴ T1 and T2 are classroom-based teacher trainers, whereas both the researchers involved, teach mathematic education in pre-service teacher education college.

observations of both of the teachers. The main part of this work is detailed in Sensevy & al (2004).

PRESENT WORK

The purpose of this paper is the confrontation of the two specific logics enacted in the teacher's action / justification and the researcher's action / interpretation, based on the teacher's action model. This research question was born out of an unexpected difference in appraisal on some teaching techniques by teachers on one side and researcher's on the other side. In this paper, the teaching episodes are selected among the existing materials from the setup described above, as particular features that need to be explained into a widest layout. We attempt to proceed in the same way as some historians (Ginzburg, 1989) using a clue-based evidentiary paradigm, to build a comprehensive reality that could not be experienced or questioned directly, because we work afterwards. After a brief description of the particular techniques encountered in T1' practice, we present T1's point of view on his techniques and then T2's analyses of T1's teaching techniques. The last part of this paper is an attempt to understand the two different systems of meaning and the two different epistemologies enacted in the different actions and discourses.

A DESCRIPTION OF TWO TECHNIQUES USED BY T1

After the analysis of the first lesson of T1, the researchers agree on the following point: two techniques used by T1 seem rather rare⁵ and unexpected. Indeed, two original ways of acting are used by T1:

- in the beginning of the lesson, he asks students to question him questions about the topic of this lesson,
- in the pair work that he scheduled for this lesson, a third student has to watch the game played by the two others as a referee.

The "questions" technique

T1 begins the lesson by questioning the students about the meaning of the words "Race to 20".

1-T1 : Today we are going to work on the race to twenty. It's a mathematical game. From the expression "race to twenty, what can you already tell me?"

2-Student : (...) we jump from 20 by 20

[...]

3-Student : Maybe we are going to count from twenty to the next twenty more.

4-T1 : Counting from twenty to the next twenty more. Yes.

5-Student : Its a race. We have to be quick...

⁵ Among the tenth of teachers that the research team studied in this situation, it was the first time that such techniques were observed.

6-T1 : *Yes. The race. The idea is to be quick. So there is velocity, since we are in a race. Or else we would have called that the walk to twenty. Maybe. Then twenty, so you say “counting twenty by twenty”. Can you see another idea?*

7-Student : *Its a race involving twenty children, with twenty children who play, who are racing.*

8-T1 : *Twenty is the number of children taking part to this. Can you see any other thing? Race to twenty. Its true that “to” may be...*

9-Quentin : *For example we are going to count three by three up to twenty. And the winner is the first to reach...*

10-T1 : *And then in that case, what is twenty ? What does it represent? Yes, go on Quentin.*

11-Quentin : *Its the number up to which we must go.*

12-T1 : *The number up to which we must go, the number we must reach. And why are you thinking of counting thee by three?*

13-Quentin : *Well, because at the moment we are working a little bit on mental counting, so that counting three by three, we learnt through going backwards.*

14-Camille : *Yes, but counting three by tree, if you start to zero up to twenty, we reach thirty but not twenty. It won't be the exact number.*

15-T1 : *You think we can reach twenty when we start from zero ?*

16-Camille : *No, with three...*

17-T1 : *Jumping three by three...*

18-Camille : *Yes, and starting from zero.*

[...]

19-T1 : *Well, in the race to twenty you suggested several things. The race, effectively, there is an idea of velocity and then, twenty, as Quentin said a few moments ago, you must reach twenty. You must go up to twenty. Another game, we can change it. So, in order to play that game, do you have enough information if I say to you “we are going to play the race to twenty”?*

20-Student : *No.*

21-T1 : *Well, in that case ask me questions!*

In contrast with the other teachers previously studied (and with T2), T1 institutes a “Question-game” (ST⁶ 1). This episode lasts almost 20 minutes in which the students try to guess what the Race to 20 is supposed to be, by considering the meaning of the “Race to 20” by considering the meaning of the words. T1 refuses gently the “wrong” answers (e.g. on ST 8). Then he summarizes the students’ answers, to emphasize the fact that the students have not “enough information” to play the game. On ST 21, the teacher produces an utterance: “So, ask me questions!”, which is emblematic of this “teaching technique”. It seems that the division of the activity between the teacher and

⁶ In the following, “ST” stands for “speech turn”.

the students is upside-down: the students ask the “relevant” questions, and the teacher give answers.

The “referee” technique

Ten minutes after the beginning of the lesson, the students are correctly playing the game. T1 organizes the group work.

39-T1 : *We go through the following steps, now you can play, since you seem to have understood. Therefore you are going to play one against one, and one child will be the referee. Because, a few minutes ago I thought there was a mistake and eventually there was none. I had not heard correctly. So, do be careful, don't try to go too fast and pronounce correctly ... sometimes it actually looks like table tennis. It goes very fast. So, do be careful about this, to be well understood. So, you play one against one and the third child is the referee. Remind me, what part does referee Arnaud's play?*

40-Arnaud : *Do the counting up (...).*

41-T1 : *So that's to give an account of the match balance. Who won? Who lost? That's the game, isn't it? Jacques?*

42-Jacques : *Does the referee writes on a sheet ?*

43-T1 : *So, for the first game, we are going to watch very accurately what is happening. But the referee plays another part as well. For instance if a child adds three. Can he do this?*

44-Student : *No, he cannot.*

45-T1 : *If a child gives several times... he says one number then he says another one... in fact one does not know any longer what he said. So, in that case, the referee supervises a little bit the respect... sees to it that the rules be respected. Yes Jacques?*

The teacher institutes the refereeing function of a third student in the group (ST 39). He defines the main features of his role. The referee will be maintained during both lessons. Many times, his task will be discussed in the whole class activity. The studying of the two lessons transcript make us conjecture that this way of acting could be a classroom habit, not specific to mathematics.

A first analysis of these techniques

The two techniques are analyzed by the team research in the same way (for detailed analyses, see Sensevy & al, 2005). Our hypothesis was that these two techniques might be counter-productive from a didactic viewpoint. Indeed, the “questions-game” could slow down the student's activity. The students' attention could be taken off the mathematical aspects of the situation. We conjectured that the “question-game” could work as a metacognitive shift (Brousseau, 1997). In a similar way, the refereeing could affect the involvement of the students in the mathematical tasks. It could also draw their attention to the superficial features of the game (the “basic rules”), and be detrimental to the production of mathematical strategies⁷

⁷ The distinction emphasized by Hintikka (e.-g. 1999) between “definitory rules” and “strategic rules” is useful : one cannot play chess successfully if one knows only the “definitory rules” (how chessmen may be moved, etc...) and does not master the “strategic rules” of this game. In the race to 20, mastering the

THE TEACHER'S POINT OF VIEW ON THESE TECHNIQUES (T1)

About the “questions” technique

When watching the video recording of his performance, T1 comments the “questions” technique. We can infer from these comments that the “questions-game” is really important in this teacher’s practice. It allows the students to find “some coherence” in the learning process. It is obvious that the teaching intentions are far beyond the didactic goal of this lesson. The “questions-game” could be considered as a taken-as-shared way of acting, that seems normative in this classroom community... It is not specifically related to the mathematical knowledge but mostly an educational technique, that could apply to any subject matter area. This technique is produced in order to fulfill some constraints of the didactic process *i.e.* make sure the didactic experience remains coherent for the students, and develop inquiry procedures in the classroom. This argument seems to corroborate our analysis of the different “division of the activity” that this technique entails. In the “question” game, the teacher’s role is not so easy: the students have to interpret the teacher’s behaviors in the right way.⁸

About the “referee” technique

The teacher’s comments make us understand that the “referee technique”, as it is enacted in the “Race to 20” lesson, is a frequently used technique (also used, for instance, in Physical Education) that the teacher applied to the mathematical pair-work designed in this situation. In other cases, the referee is said to be useful for the evaluation tasks of the knowledge, but the teacher admits that criteria for assessment are not easy to define. This is interesting because the teacher reveals himself that these technique may not fit with all the class activities.

A first interpretation

In order to understand the teacher’s action in this lesson, and particularly in the management of the two techniques that we showed, one has to consider the function of these techniques in the teaching process. T1 is concerned in creating an inquiry-based classroom, and possibly to delegate the assessment task to the students themselves, and this, not only in mathematics or in science, but in a general way, in all the classroom activities. In order to create such a self-directed learning, the teacher calls in some general techniques that can be *replicated* from one situation to another, which bring some coherence in the learning experiences. A didactic analysis make us conjecture that these techniques are not very efficient from a mathematical viewpoint. Nevertheless, the role of the teacher, in primary school, is not only to foster the mathematical thinking and sense-making of the students: it is also to educate them, to

“strategic rules” implies for instance the discovery of such a “rule” as “17 wins, so the “Race to 20” equals to the Race to 17”.

⁸ It is interesting to notice that the teacher attempts to apply it to the “Race to 20”, where apparently, there is no relevance for links to be found. The mathematical situation is dropped by the researcher in the teaching process of this class, without any peculiar connections with the subjects studied previously.

give them “cognitive values” (Putnam, 1992) embedded, for instance, in self-questioning or inquiry process.

THE OTHER TEACHER’S (T2) ANALYSIS OF THESE TECHNIQUES

About the techniques

When watching the videotape of T1’s lesson, T2 shows interest in the “questions-games” episode.

1.T2 .. *That’s the point I wanted to ask you there. You... you... ask the pupils to speak?*

2.T1 *I wanted to.*

3.T2 *So this means... yes.*

4.T1 *I wanted to start from the race to twenty since... showing people... so there it was a questioning to see which meaning they could give when there was a new discovery.*

5.T2 *Yes I found that it was interesting exactly because they were taking part, I would not have thought at the beginning at first, but I found that it was worth doing it.
[...]*

6.T2 *When you were having this questioning there, in fact, you felt that it should create links, or was it because you intended simply to explicit the vocabulary?*

7.T1 *Er... no, it was in fact, looking for meaning. Starting from an idea, well, from a proposition, an expression, different meanings, to be able to rebound afterwards a little bit later on. Er... Now we can say that it was rather that way. But that might have been something else. [...]*

10.T2 *Yes but. Your asking a question. Er... On the meaning, and the children. Er... giving an interpretation linking it to something else, I find that, for me, it’s interesting.*

In his comment, T2 grasps T1’s intentions. Notably in ST 6 and 10, T2 stresses that “making links” is important... T2 seems to recognize some “valuable” features in these techniques, that may corroborate our hypothesis of a generic constraint about connections between tasks, that lies upon teachers.

T2 synthetic commentary

T2 is then asked to give some conclusive comments on T1’s practices :

T2 *About the session, in fact, I notice that we did not at all take the same beginning. There are things which I would never have thought about because I don’t practice them in my class... In fact it gives me ideas, you know, I will try some things. I really enjoy the part of the referee coming from outside, because I do it as well among the groups but it is always within the groups (...) And then there is one thing which I will keep in mind as well: the way you get in the activity with your insisting on the language, the meaning of words ; that I find maybe a way to start. That can be done. That I can do with other activities. But I would never have thought about it, there, for example, and I find it is quite right when starting an activity, to make a link or to avoid disconnections with previous sessions.*

In these comments, we can find arguments that expose very clearly the epistemological gap between the teachers and the researchers. Indeed, T2 appreciates the referee as an outsider. In contrast, researchers analysis show how this referee could be a *mathematical outsider*, who does not mathematically benefit from the situation.

Similarly, T2 emphasizes, in a very direct way, two fundamental functions of the "questions" technique : to link different activities *a priori* separated ; to avoid the temporal break between the different lessons. There is no consideration for the mathematical content at stake⁹. Finally, the gap is obvious, between researchers who are primarily concerned with the specific mathematical meaning of the situations, and teachers who are primarily concerned with the coherence of the classroom activities and the educational relevance of *replicated forms* of teaching actions.

TWO DIFFERENT SYSTEMS OF MEANING, TWO DIFFERENT EPISTEMOLOGIES

These two systems can be described as following. The "researchers system" is oriented towards the mathematical content, enacted in specific mathematical practices. The Race to 20 is a mathematical situation that includes several prominent features: the "alternative" (if I play 17, either my opponent plays 18, or he plays 19); "the backwards recurrence" (to play 20, I have to play 17, therefore the race to 20 is a race to 17; to play 17, I have to play 14, therefore the race to 20 is a race to 14... the race to 20 is a race to 2) ; the "methodological" triplet *proof-conjecture-refutation*.. For the researchers, the appropriate didactic contract (Brousseau, 1997) contains these objects, but the teaching practices of T1 and T2 do not include them. Following the distinction coined by Cobb & al (2001), we could say that the researchers put into focus the "mathematical practices" in the classroom. On the contrary, the "teachers system" is based on the generic relevance of some teaching techniques. The appropriate categories are "the development of the student's autonomy", and "the necessity, for the students, of assuming their learning responsibilities"... The generic teaching techniques bring the coherence in the didactic experience of the students. The priority lies, therefore, in fostering "social norms" (Cobb& al, *ibid*) in the classroom. As we find important to explore the ways in which the gap might be bridged, we chose, at least in a first study, to put on hold our theoretical stance¹⁰. In doing so, we took the opportunity to understand the practical logic of the action. Now, in order to organise the discussion of analyses, we shall come back to the levels of the model of the teacher's action, that we introduced at the beginning. We hypothesize an *overdetermination* of the third level, in which cognitive values and teaching practices are embedded, upon the two *infra* levels, at least at primary school. The generalist task of the primary school teacher may foster this overdetermination, compared with secondary school teachers who are usually in charge of a single subject. However, even within a same subject, we cannot *a priori* minimize this phenomenon. In the same way, some didactical techniques appropriated to teach an arithmetic knowledge may turn to be counterproductive to deal with some geometrical knowledge.

⁹ In a more general way, there is very little care, during the teachers' dialogue in the cross-analysis, for the discussion of the mathematical knowledge.

¹⁰ In doing so, we try to avoid the effects of the «scolastic fallacy» (Bourdieu, 1994, p. 112) when social scientists "think that agents involved in action, in practice, in life, think , know, and see... as the scientist whose mode of thought presupposes...distance and freedom from the urgency of the practice"

If we want to understand the teacher's action, we have to document how the cognitive values, teacher's beliefs and practices can shape the didactical transactions. The techniques used by the teachers appear to be answers to educational constraints (e.g., "try to establish a link between a maximum of activities that, on the face of it, don't have anything to do with each other" or "the assessment tasks must not rely only on me, the teacher"). However, a technique that the teacher replicates in a general way in different activities, could be the answer to a *generic* constraint, from the didactical point of view. Indeed, the didactical theory (Brousseau 1997) shows that pieces of knowledge have intrinsic links between each other. The mathematical situations are designed to help the student in building bridges between different pieces of knowledge. However, the links between situations, or class activities are not obvious and often depends on an *a priori* knowledge organisation made by institutions (curricula, textbooks). Therefore the teacher has to cope with the situations, trying to replicate some teaching techniques, but not very specifically related to the mathematical knowledge, in order to reach an educational achievement. Meanwhile some *generic* didactical constraints exist about the knowledge organization and could be playing in the background. Therefore conflicts may emerge between the teaching techniques and the didactical goal of a mathematical situation, as we saw in T1's lesson. This explanation induces that the teacher may need some specifically knowledge-related techniques, to meet both didactical and educational achievements. Mastering the mathematical content knowledge is of course essential but not sufficient. For example, we can take for granted that T1 is familiar with the Euclidean division through the Race to 20 because he had a three hours mathematical training on this, provided by the research team. However he did not call in the specifically knowledge-related techniques in the classroom (e.g. the *alternative* technique : "if I play 17, the other can only play 18 or 19, and I reach 20" or the *recursive* technique : "17 wins, so the "Race to 20" equals to the Race to 17") that enable the students to identify eventually the Euclidean division in the game. A technical gap has to be overcome between the mathematical content knowledge and knowledge-related techniques.

To conclude, we think that the researchers have to understand the very nature of the teacher's action. That means to identify the different constraints that the teachers have to cope with, in particular the necessity of *educational coherence* for the teachers which that can be explained by some *generic overdetermination, from the didactical point of view*, that we conjecture in this paper. Against such a background, a collaborative research could allow the researchers to acknowledge the multi-determination of the practical logic, and the teachers to analyze the mathematical content in a more efficient way, the collaborative research attempting to answer the following questions: what could be the *specific teaching technique* that the teacher has to produce to foster the mathematical thinking and sense-making of the students? What could be the effective conditions of their productions?

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THE ROLE OF THE PRACTICE OF THEORISING PRACTICE

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Abstract: *This paper consists of a set of reflections on a development programme for teacher education in Eritrea. I shall frame the Eritrean story by considering the relationship between theory and practice more generally. I shall, then, first describe this relationship theoretically and argue that it may be described as a theoretical loop, starting in and returning to practice. Second, I shall adopt a rather more practical perspective and describe how theoretical mini-loops may be used to inform curriculum change, in this case in Eritrean teacher education. Third, I shall use the metaphor of the theoretical loop when suggesting what other conditions of possibility are required, if teacher education is to significantly inform the teaching and learning of mathematics in schools. I shall build on an earlier Danish study and on the Eritrean example to make the point.*

Keywords: theory and practice; curriculum change; actual and virtual communities of practice; cracks and openings in the enacted curriculum.

EMPIRICAL RESEARCH, THEORY AND PRACTICE: A THEORETICAL LOOP

The relationship between empirical classroom research and teacher education has changed. Theoretical constructs to conceive of the role of the teacher as well as practical suggestions for teaching and for supporting teacher development are now based on studies of the interactions of mathematics classrooms. For example Cobb et al. (1997) suggest how the teacher's use of a symbolisation of solutions to a task may become the means to shift the attention from the initial question to meta-concerns of whether all possible solutions have been found. This suggests that students in teacher education work with how locally derived symbolisations may facilitate such meta-cognitive shifts. Jaworski (1994) developed the notion of the teaching triad as a description of how one may balance three interdependent domains of teaching activity, the ones of managing students' learning, reacting sensitively to their needs and challenging them mathematically. Later Potari and Jaworski suggested using the teaching triad as a reflective tool in teacher education (Potari & Jaworski, 2002). Skott (2001) used the notion of critical incidents of practice (CIPs) to describe classroom interactions in which multiple motives of the teacher's activity emerged and which both challenged the teacher's school mathematical priorities and were critical to the students' learning. He has also suggested using CIPs from the student teacher's own practice as focal points in teacher education (Skott, 1999).

These studies exemplify that mathematics education has become an empirical field aiming to understand the learning opportunities emerging in the classroom. Each of the studies goes through a theoretically informed process of theorising practice, i.e. one of developing constructs that contribute with new ways of understanding issues emerging from the interactions of the classroom in question. In turn, new theoretical constructs gain at least part of their legitimacy from their ability to guide instruction. Cobb terms this a “reflexive interdependency” between theory and practice:

[...] the theoretical issues we addressed emerged from our practice of working with teachers and students. Further, our attempt to develop a theoretical alternative was guided by images of classroom practice and fed back to shape that practice. (Cobb, 1995; p. 240)

This is opposed to a field that is primarily concerned with analysing mathematical contents and with conducting clinical analyses of students’ work on such contents. In other terms, mathematics education has moved from a development-implementation conception of curriculum in which a priori analyses of content formed the basis of subsequent attempts to implement reform. Reflexive interdependency points to the interconnectedness of theory and practice and suggests viewing a possible split between the two as one of the false dichotomies of mathematics education, although ‘false’ is interpreted in pragmatic, rather than ontological terms.

Partially, the changes outlined above are in tandem with developments in curriculum theory. Schwab noted 20 years ago that one should not expect a priori theoretical constructs to significantly inform teaching practices or student learning: “A linear movement from theory to practice is absurd” (Schwab, 1983, p. 241). Since then, the sections of curriculum theory concerned with the practices of education have replaced or supplemented the theory-into-practice approach of the traditional field with what is essentially the opposite movement, i.e. with attempts to understand the learning potentials that unfold in classrooms: Theorising practice has become a major concern.

Contrary to the reconceptualised curriculum field (e.g. Pinar et al., p. 8), most research in mathematics education is highly committed to practice, i.e. to developing understandings that may feed back into the classroom. The theoretical orientation in mathematics education, then, does not turn the field into an exclusively theoretical one, and theorising practice has become one of the most significant ways of attempting to overcome a traditional split between theory and classroom teaching. To the extent that this attempt is successful, it replaces any notion of linearity between theory and practice (cf. the quotation from Schwab), with a theoretical loop, beginning in practice and potentially providing input to practice, not in terms of prescriptions for teacher behaviour, but as focal points for or frames of reference of the teacher’s reflective activity (Skott, 2004).

It is reasonable to assume that theoretical constructs grounded in the sites of practice are of greater potential use to practitioners (e.g. teachers) than constructs developed

without such grounding and without recognition of the contextual complexities of teaching. However, classroom teaching is directed at other concerns and interests than theorising. There are, then, two sets of distinct practices involved in Cobb's notion of reflexive interdependency. First, it concerns the practices of teaching and learning and the relationship, which may be informed by theoretical constructs developed beforehand. Second, it also involves *developing* the theoretical constructs, i.e. the practice of theorising. Reflective teaching, teachers' inquiry into their own practice, and large amounts of action research indicate that one may be committed to both these types of practice at the same time. And the last few words in the quotation from Cobb make the point that the results of the latter type of practice may significantly inform the former. In spite of this, reflexive interdependency does not map the practice of theorising neatly on top of the practice of teaching: Even if conducted by the same people and mutually framing and constituting one another, these are not *the same* practices, or at least they have different emphases.

In the next section, I shall describe a teacher education development programme in Eritrea. The programme is rhetorically inspired by the results of theoretical loops dealing with social interaction, communication and the role of the teacher. Within the programme, mini-studies were conducted to guide more specific suggestions of the reform. For instance, mathematics teaching in elementary schools was studied to develop understandings of how new modes of interaction may be developed and sustained under contextual constraints of for instance very large classes. The suggestions made may be seen as the outcome of theoretical mini-loops. The question remains as to what extent these suggestions address the problems of practice as conceived by the prospective teachers, even if they do become part of the enacted teacher education curriculum.

REFORMING ERITREAN TEACHER EDUCATION IN TIMES OF CONSTRAINTS

Eritrea and the official educational discourse. Eritrea is a small developing country that gained her independence in 1993 after a long liberation struggle against Ethiopia. Inhabiting the horn of Africa, the 3½ million people from 9 different ethnic groups speaking 9 different languages live under harsh conditions. For example, per capita gross national income is a meagre US \$ 250, infant mortality rate runs at more than 10%, and life expectancy is as low as 46. Literacy rates are 10% and 20% for women and men respectively, and net enrolment rate in elementary school is 45, i.e. the proportion of children of elementary school age who is enrolled in school is 45%.

Comprehensive plans for a transformation of Eritrea's education sector have been developed and external funding has been found to support it. To a large extent, the discourse related to the present situation and to the proposed reform is cast in terms of the educational rhetoric of the West. For instance, a situation analysis of Eritrean education calls for coherence and integration of subjects as well as interaction and

communication to replace ‘passive listening’ and ‘didactic and traditional pedagogy’ (Ministry of Education, 1997). Similarly, Osman Saleh, Minister of Education, says in the recommendations to guide a reform of the school curriculum that

A learner centered and interactive pedagogy is central to the New National Curriculum [...]. This is guided by the principle that learning with understanding is an active and participatory process. Effective learning occurs when students are interacting with one another and the teacher. [...] Our commitment to learner centered and interactive pedagogy places a heavy responsibility on our teachers [...] to involve students in the process of generating essential knowledge and skills. (Osman, 2003).

The Eritrean educational discourse, then, resembles its international counterpart. However, contextual factors limit the opportunities for the rhetoric to play prominently in practice. This is so to a greater extent than e.g. in the West. The main problems of what is termed *the ground situation* are poor preparation of teachers, large classes, lack of quality materials, and poor physical conditions¹.

A programme to reform teacher education: At present, teacher education for the elementary grades is a one-year programme, formally qualifying the graduates to teach all subjects in grades 1-5. According to official figures, 73% of Eritrean elementary teachers are formally qualified (Ministry of Education, 2002).

As elsewhere, it is an urgent educational priority to provide quality education for teachers in line with the intentions of the intended school curriculum. In 2001 a development scheme was introduced to revise curricular materials and change the teaching-learning practices of the present teacher education course. The course, however, is still to qualify the graduates for teaching all subjects in just one year.

The initial intentions of the reform were vague, but in line with the ones indicated above. In particular, the new programme was to prepare student teachers for increasing their prospective students’ participation in the classroom. As part of the development process, Eritrean teacher educators and a Danish consultancy team jointly turned the intentions into explicit visions of teachers and of Eritrean teacher education. The visions are informed by the results of a situation analysis of the contexts in which the new teacher education curriculum is to function. This analysis was also to qualify both teacher educators and consultants for the task of developing such a curriculum. In the following, I shall discuss the part of the analysis on mathematics. This was done by two Eritrean teacher educators and a consultant, the author of this paper. Whenever I write ‘we’, I refer to the three of us.

The situation analysis consists of (1) a review of existing curricular documents for teacher education; (2) discussions with teacher educators about the present

¹ This list of the most significant aspects of ‘the ground situation’ came out of a workshop in the curriculum department in the Eritrean Ministry of Education in 2003.

programme and their professional problems; (3) a mini-study of the students' educational and other background; (4) interviews with students about the present programme and their expectations for their future professional life; (5) observations of college classrooms; and (6) observations of elementary classrooms with follow-up interviews with students and teachers.² I shall describe the last of these in more detail.

Observing elementary mathematics. In the spring of 2002, we individually visited six schools in different parts of the country. At each school, we observed 2 teachers teach 2-3 lessons each. The intention was not to collect data that were considered representative. Neither was it to ground all our suggestions for the teacher education programme in the practices of elementary school. Rather, it was to develop some understanding of the perceived problems of Eritrean elementary teachers soon after their graduation. More specifically, we sought to understand the problems with encouraging other modes of communication and student participation than those depicted in the common discourse on the present state of affairs: the teacher as explicator of concepts and skills and the students as 'passive listeners'. To the extent that such understandings provoke or resonate with those of the mathematics educators, they may suggest possible routes for curriculum change, routes which need to be explored further and may then be used to inform curricular decision-making.

Methodological problems and solutions. Methodologically, the observations posed significant problems: we all observed classes taught in languages that we do not understand. This is a particular problem, as the focus was on the communication and activity encouraged and sustained. Also, the two teacher educators had no prior experience with qualitative research. However, after a short seminar on qualitative methods and some introductory reading, we jointly developed an observation schedule, combining structured observations with more qualitative ones.

The schedule has three parts. First, it focuses on physical and other immediate characteristics of the situation. These include the number of male and female students, the position of the desks, the availability of textbooks, etc. Also, the position of the teacher every five minutes is to be shown on a sketch of the classroom. Apart from the last point, this first page of the schedule may be filled in before the lesson. Second, the observation schedule includes a table with horizontal time lines and vertical headlines of *teacher activity*, *student activity*, and *classroom organisation*. Focussing on character of the communication, the listed teacher activities were *posing open questions to boys/girls*, *posing closed questions to boys/girls*, *giving negative feedback to boys/girls*, *giving positive feedback to boys/girls*, *providing information/lectures*, and *other*. Suggested student activities

² One element is conspicuously absent in this list of activities: a review of materials for the school curriculum. However, a revision of this curriculum was underway, and we were not allowed to view the progress being made, let alone be part of the revision process.

were *listening, reading, solving routine tasks, and solving problems*. Suggested types of organisation were *whole class, group work, pair work, and individual work*. The third page of the schedule was to be filled in after class and dealt with the *Overall impression*, and invited an open description of the perceived student-teacher relationship, classroom management and mastery of content. Minor modifications of the schedule were made as needed in the process of making the observations.

After an observation, but on the same day, the observer was to write a more coherent and qualitative account of the observations. To illustrate this, the first lines of the account of Abdul's teaching in grade 2 read like this:

The topic of the class was multiplication, or rather the 1-times table. Before getting down to business they enthusiastically sang a song in Arabic. After that Abdul writes the following on the board, and when he finishes, he says "1 by 1 is" and the students all reply while he points to the answer. They go over it all. The children pour a lot of energy into it all at this stage. (1-7 min.).

$1 * 1 = 1$	$1 * 7 = 7$
$1 * 2 = 2$	$1 * 8 = 8$
$1 * 3 = 3$	$1 * 9 = 9$
$1 * 4 = 4$	$1 * 10 = 10$
$1 * 5 = 5$	$1 * 11 = 11$
$1 * 6 = 6$	$1 * 12 = 12$

Also on the day of the observation, the teacher was interviewed about the lesson and about her work more generally. Also during the interview, particular questions were discussed that arose from making the observations, from filling in the schedule or from transforming the schedule into a more qualitative account. In practice, the qualitative account became the basis of the subsequent coding.

Abdul's school and classroom. I visited a school in the Western lowlands, bordering the Sahara desert. The school, a brick building of five classrooms, was recently rebuilt after having been destroyed during the struggle with Ethiopia. This is where Abdul teaches. Abdul is a college graduate in his mid 20s. He has taught at the school for 18 months. Currently he teaches mathematics to a grade 2 of app. 50 students.

Following the introduction to the lesson (cf. the quotation above), Abdul rewrites some of the tasks on the board, without deleting what he already wrote. This time he writes the tasks without the answers and in a different order. He asks the students to come to the board one at a time. The students are enthusiastic and keen to be selected. Everyone claps their hands, when a student finishes a task. When all the tasks have been done, Abdul repeats the first activity of going over the table, which is still written on the board. The students are chanting, Abdul is pointing to the answers. He then writes more questions from the 1-times table, and they deal with them in a whole class setting. The students get out their notebooks and begin working on 9 tasks that Abdul writes on the board ($1*5 =$; $1*8 =$; $1*6 =$). He wanders around the class, but does not interact with the students. Half an hour into the lesson, Abdul asks the students to put their notebooks away, but at first he does not initiate a new activity. Gradually the enthusiasm and energy dwindle. In the end Abdul goes over the table once again, but by now he has lost the momentum.

Abdul's teaching and that of other teachers. A number of aspects of Abdul's teaching were discussed further as possible sources of inspiration for teacher education. One aspect was that Abdul's classroom did not fit the common description of students as passive listeners. On the contrary, the students were generally fully engaged with the tasks. They were listeners, in the sense of not being involved in setting the tasks and of being told exactly how to solve them. But they were eagerly involved, waving their hands in order to present solutions. This is especially so in whole class settings in the first half of the lesson, but also when they worked individually on similar tasks.

The students in the other classrooms visited showed similar enthusiasm. According to both teachers and students this was not special in comparison with everyday teaching, when there were no observers. It is worth noticing that at least in some contexts, it is possible to capitalise on different modes of classroom communication than those exclusively referring to students as 'passive listeners'³.

There are two other issues that deserve attention and that suggest more immediate changes in teacher education. The first is the students' *restricted mathematical participation*. Although the students are very active, the types of activity they become involved in, are extremely limited. The questions raised are always preceded by clear instructions as to how they should be addressed. Sometimes the students are to repeat the teacher's explanation orally or in writing. At other times they are to copy a procedure, replacing a number when doing so. In both cases, the students' are only to perform procedures that have been explicated before. The only exception is when they are asked to solve tasks that require them to combine two or more of the tasks already solved. For instance, a teacher implicitly asked the students to combine their previous activities on perimeter and area of squares by asking the questions like *The area of a square is 196 m^2 . Find the perimeter of the square.*

This led us to formulate the intention in primary school of *extending students' mathematical participation*.

Second, Abdul's effort to cover the 1-times table is striking. I notice that some students have already done work on other multiplication tables in their note books. In the interview, Abdul explains that he has taught the multiplication tables before, but the students found the 8-times table hard. That is why he returned to an easy example.

This approach is similar to that of Abdul's colleague, Omar. Omar teaches addition of fractions in grade 4, but his students find it difficult. For instance, they have problems with the first of the following tasks. Addressing their difficulties, Omar writes the second task immediately below the first one:

³ One of the differences between Eritrea's ethnic groups is their views of children's right to speak out. It is possible that a less inviting classroom atmosphere is found in other areas.

$$(1) \quad 2/3 + 2/3 + 2/3 = 3 \text{ mednotta } 2/3 = 3 * 2/3 =$$

$$(2) \quad 1+1+1+1 = 4 \text{ mednotta } 1 = 4*1 = 4$$

In these instances the teacher attempts to resort to well known contents in order to solve students' learning difficulties. This is a laudable approach, when interpreted as an attempt to adjust one's teaching to the students' present understanding. Also, it is a piece of methodological advice that is consistently made in the teachers' pre-service education. In the cases above, however, the teachers chose the easier task because of its apparent structural similarity with the more difficult one, rather than because it addressed the students' learning difficulties.

These and other similar experiences led us to formulate an intention for primary school of *instantaneously interpreting students' understanding*, well aware of the problems this poses in classes of 50 students or more.

These two issues resonate with other parts of the situation analysis, especially with the observations of college teaching. For instance, communication in college classrooms often limits the students' participation in much the same sense as in Abdul's teaching. Questions that are apparently open are often asked in teacher education, both on the mathematical contents and on methodological issues (e.g. *What do you know about fractions? How do you think an elementary school child would do this?*). However, more often than not, questions like these turn out to be closed in the sense that only one or a very limited range of answers or solution strategies is accepted. Consequently, they involve the students in a guessing game with regard to what the teacher considers the only acceptable answer. The teacher educator's response, then, is one of a mathematical or methodological closure, i.e. one that does not encourage the student teachers to become involved in the types of interactions and communal inquiry that is called for in the visions of the new school curriculum. This closure also suggests that the teacher educators do not engage in an activity of probing and interpreting the understandings of their students. This is particularly important as modelling good teaching from the beginning was considered a necessary, but insufficient characteristic of quality teacher education.

THE ACTUAL AND VIRTUAL COMMUNITIES OF A NOVICE TEACHER'S PRACTICE AND THE CRACKS AND OPENINGS IN THE SOCIAL FABRIC OF THE ENACTED CURRICULUM

Actual and virtual communities of practice: I have suggested elsewhere in relation to Danish novice teachers that teaching must be viewed as an activity that is constituted by the context in which it unfolds (Skott, 2002). This challenges the extent to which it makes sense to talk about a teacher's practice in a possessive sense. At first sight this may also seem to question the role of teacher education, when it is located in colleges, temporally and geographically distant from the classrooms in which the

students are to function. This would be so, even if the contents of the programmes are the results of theoretical loops, grounded in practice and intending to inform practice.

However, the professional context of the Danish novice teachers did not primarily consist of the community of teachers at the school. Indeed, they referred more frequently to their teacher education background than to their colleagues when addressing problems of managing classrooms, supporting students' learning, and developing teaching-learning materials. The teachers' identity, then, as facilitators of learning was constituted not primarily at the school, but in their continued participation in a virtual, and probably fading community of teaching practice with the teachers and peers of their pre-service education.

Abdul and his colleagues also relate to their pre-service education as the context for their educational discourse. In the interviews, they refer to their college course when explaining their classroom activity. Abdul particularly refers to "the methods of teaching" as the most significant aspect of the course.

However, the Eritrean teachers also mention their colleagues and especially the headmaster as significant for their educational thinking and practice. Every week they need to have their lesson plans accepted by the head, who is also the person to turn to for advice. The teachers speak positively of this sanctioning of the lesson plan, as well as of the role of the other staff. For instance Abdul claims:

Without the plan I can't do anything. The director gives good advice. He suggests real objects. These are rural children. They don't understand, if they are not shown [...] [In staff meetings] we discuss the weaknesses of the students, are they from us from the students? Are we using different methods [...] If I am teaching in one method, maybe they do not understand. The director has told [us] to use different methods. (Abdul, the interview)

It seems, then, that both the actual and virtual contexts of Abdul's teaching have significant impact. It was beyond the scope of the situation analysis to investigate the degree of compatibility and the relative strengths of these contexts for his practice.

The cracks and openings of the enacted curriculum: Most frequently, theoretical loops are used to develop the very practices that are researched. For instance, a study of multiplicative reasoning in grade 4 may suggest novel ways of teaching it. But even in this case, it would be ironic to expect the results of a theoretical loop to have an immediate impact in classrooms. After all, the development-implementation approach to curriculum was replaced primarily because of the need to include contextual factors that challenge the linearity of 'theory-into-practice'. But even though - or exactly because - a loop takes a specific set of contexts as the starting point, it does not relate equally well to all contexts. It would then be contextually naïve to expect the outcomes of a theoretical loop to fit smoothly with the social fabric of any school and classroom.

However, things are complicated further, when studies of school classrooms are to inform the practices of teacher education, which in turn are expected to contribute to change in schools. In this case, the theoretical loop does not return to where it started, but to a different practice, i.e. to the one of the teacher education programme.

Theorising and teaching are not the same practices (cf. section 1). Similarly, the practices of studying to become a teacher are not the same as the ones of teaching. For instance, learning that there is a need to extend the students' mathematical participation or even learning how that may be done within the context of teacher education, is not the same as being able to do it in school upon graduation.

The results of a theoretical loop, then, do not necessarily become part of the practices of an elementary school, even if they are reflected in a new teacher education course. The extent to which this is the case depends on the compatibility with the structures and traditions of the school and on the cracks and openings in the structure. For instance, Abdul's teaching is distant from any intention of extending the students' mathematical participation. However, the types of interaction and student enthusiasm that prevails in his classroom provide an opening that may be used to insert other types of mathematical activity than the ones that now dominate his teaching. If teacher education programmes are able to establish virtual communities of teaching practice along the lines of the reform, they may be able to infuse reform intentions into such cracks and openings. In other terms, reflexive interdependency does not ensure that the results of theorising contribute significantly to the practices of teaching and learning. Communal reflection is needed that links and correlates the results to the practice of teaching within the context and community in which it is to be conducted.

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VIEWS ON EARLY SCHOOL MATHEMATICS – EUROPEAN NEEDS AND THE LOCAL REALITY

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Abstract: *Certain phenomena that take place in school can be anticipated, taking into consideration functioning of causal-efficacious dependences. Such relation exists between the image of mathematics as a school subject existing in teacher's consciousness and a way of realisation of the subject in practice. In this article I present the results of recognition of the actual image of mathematics conducted in a group of 118 Polish students.*

Keywords: teacher's attitudes, a survey.

The teacher will always be a central figure that is responsible for all that is going on in the classroom during maths lessons. *Given the important role of the teacher in the educational process, it appears quite natural to study in-depth his or her personal philosophies about mathematics* (da Ponte, 1999). On the stage of preparation for being a teacher it is a recognition of philosophy of mathematics that the student possesses.

Students who decided that their future occupation would be work with a child, begin studying with a certain image of what mathematics is, as a school subject, and what the teacher's work on maths lesson looks like. Their knowledge is not a professional one but can impinge on what degree the theoretical teacher's background will be accepted by them. The learning student and future teacher will be mainly focused on gaining knowledge that, to his mind, will have application in practice. On an academic level of studying, general rules of studying are in force and according to this *'recognition processes are strongly connected with the whole human activity... Undertaken and executed tasks must occur to themselves as important and useful.* (Piotrowski, 2004, p. 170).

I have been examining the attitude of teachers and students to school mathematics during 2003 and 2004, mainly among part-time students. The core of the group consisted of 118 part-time students of Pedagogical Faculty Rzeszów University. Those were the future teachers of early mathematics (children aged 6-9) whose occupational preparation consisted of: language, mathematical, biological and artistic, technical and physical education. After graduating from university these students will gain qualifications to work with children also during maths lessons - so students in their schedules had a few hours of mathematics or selected issues in field of methodology of teaching mathematics.

I prepared a survey with questions mainly concerning views on school mathematics, and also actions of teacher on maths lessons.

One of the questions was as following:

Please describe your attitude towards following opinions:

- a) in mathematics, the main part of material must be learnt in a way described in the book
- b) mathematical concepts are linked one to another and learning every single thing is not necessary
- c) in studying mathematics similarities help, ex between properties of sums and properties of products
- d) in studying mathematics the most important thing is to know methods of task solving
- e) during maths lessons everybody can be active
- f) mathematics is a science for the chosen ones
- g) in mathematics we always get certain answers to all questions
- h) creation of mathematics consists in finding general rules for single facts
- i) teacher of mathematics should always explain and show everything clearly, ex. how to add while crossing decimal threshold
- j) everyone creates mathematics independently for one's own self
- k) in mathematics, student can independently create the ways of solving tasks, ex. ways of multiplication of two-digit numbers in mind
- l) in mathematics everything must be done according to certain rules
- ł) mathematics was created by abstraction and generalisation. In this way, ex. the notion of number was created
- m) in mathematical task there is no place for variety of interpretation
- n) the source of basic mathematical concepts is the world surrounding us and our actions in it, ex. objects from the environment are the source of geometrical notions
- o) mathematical notions (ex a notion of a straight line) have no connection with the reality
- p) on maths lessons every student should and can think independently
- r) in school mathematics nothing is discovered independently.

The evaluation of proposed statements about mathematics and its teaching was to show preferences in attitudes: constructive and formal.

The survey shows that the students are generally nondescript unclear and do not accept any of views, or and in different situations they behave ambivalently (so - inconsequently).

Their attitude is incoherent - in many situations they contradict themselves, what may mean that when keeping such an attitude in their work they will send the child contradictory signals about their own expectations towards child's behaviours.

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